(xiii) $G = Q \leftthreetimes K$, *where* K *is a quasicyclic* p*-subgroup,* $Q = C_G(Q)$ *is an elementary abelian* q*-subgroup,* p*,* q *are primes,* $p \neq q$*,* Q *is a minimal normal subgroup of* G*.*

Contact information

Mykola N. Semko
Department of Mathematics, University of State Fiscal Service of Ukraine, Irpin, Ukraine
Email address: dr.mykola.semko@gmail.com,

Liliia V. Skaskiv
Department of Mathematics, University of State Fiscal Service of Ukraine, Irpin, Ukraine
Email address: lila_yonyk@ua.fm

O.A. Yarovaya
Department of Mathematics, University of State Fiscal Service of Ukraine, Irpin, Ukraine
Email address: yarovaoa@ukr.net

Semiscalar equivalence of one class of 3-by-3 matrices

Bohdan Shavarovskii

Let a matrix $F(x) \in M(3,\ \mathbb{C}[x])$ have a unit first invariant factor and only one characteristic root. We assume that this uniquely characteristic root is zero. In [1], the author proved that in the class $\{PF(x)Q(x)\}$, where $P \in GL(3,\ C)$, $Q(x) \in GL(3,\ \mathbb{C}[x])$ there exists a matrix

$$A(x) = \left\| \begin{matrix} 1 & 0 & 0 \\ a_1(x) & x^{k_1} & 0 \\ a_3(x) & a_2(x) & x^{k_2} \end{matrix} \right\|$$

(notation: $A(x) \approx F(x)$), which has the following properties:

(i) $\deg a_1 < k_1$, $\deg a_2,\ \deg a_3 < k_2$, $a_2(x) = x^{k_1} a_2'(x)$, $a_1(0) = a_2'(0) = a_3(0) = 0$;

(ii) $co \deg a_3 \neq co \deg a_1,\ co \deg a_2'$, if $co \deg a_3 < co \deg a_2$;

(iii) $co \deg a_3 \neq 2co \deg a_1 + co \deg a_2'$ and in $a_1(x)$ the monomial of the degree $2co \deg a_1$ is absent, if $co \deg a_3 \geq co \deg a_2$.

Here $co \deg$ denotes *the junior degree* of polynomial. The purpose of this report is to construct the canonical form of the matrix $F(x)$ in the class $\{PF(x)Q(x)\}$. If both elements $a_1(x),\ a_2(x)$ of the matrix $A(x)$ are non-zero, then we may take their junior coefficients to be identity elements. In the opposite case, we may take the junior coefficients of the non-zero subdiagonal elements of the matrix $A(x)$ to be one. Such matrix $A(x)$ in [1] is called the *reduced matrix*. In this report we consider the case, when some of the elements $a_1(x),\ a_2(x),\ a_3(x)$ of the matrix $A(x)$ are equal to zero and at least one of them is different from zero.

Theorem 1. *Let in the reduced matrix* $A(x)$ *the conditions* $a_i(x) \neq 0$*, for some index* i *from set* $\{1,\ 2,\ 3\}$ *and* $a_j(x) \equiv 0$ *for the rest* $j \in \{1,\ 2,\ 3\}$*,* $j \neq i$*, be fulfilled. Then* $A(x) \approx B(x)$*, where in the reduced matrix*

$$B(x) = \left\| \begin{matrix} 1 & 0 & 0 \\ b_1(x) & x^{k_1} & 0 \\ b_3(x) & b_2(x) & x^{k_2} \end{matrix} \right\|,$$

the element $b_i(x) \neq 0$ *does not contain* n_i*-monomial,*

$$n_i = \begin{cases} 2co\deg a_i, \ i = 1, \ 3, \\ 2co\deg a'_2 + k_1, \ i = 2, \end{cases} ,$$

$b_j(x) \equiv 0$. *The matrix* $B(x)$ *is uniquely defined.*

References

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Contact information

Bohdan Shavarovskii
Pidstryhach Institute for Applied Problems of Mechanics and Mathematics National Academy of Sciences of Ukraine, L'viv, Ukraine
Email address: bshavarovskii@gmail.com

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On greatest common divisors and least common multiple of linear matrix equation solutions

Volodymyr Shchedryk

Investigation of linear equation solutions has a profound history. Due to applied and theoretical problems we need to find roots with certain predefined properties. Matrix equations were studied with a symmetry condition, with Hermitian positively defined condition, with minimal rank condition on the solutions.

Let R be an associative ring with $1 \neq 0$. A set of all solutions of the equation $a = bx$ in R is $c + Ann_r(b)$, where c is some root one,

$$Ann_r(b) = \{f \in R | bf = 0\}.$$

Such a description of the roots is not always convenient. We would like to have their image in the form of a product. In this connection, the question arises search for the generating element of this set.

Let A, B be a matrices over ring R. If $A = BC$, then A is a right multiple of B and B is a left divisor of A. If $A = DA_1$ and $B = DB_1$, then D is a common left divisor of A, B; if, furthermore, D is a right multiple of every common right divisor of A and B, then D is a left greatest common divisor of A, B.

If $M = NA = KB$, then M is a common left multiple of A and B, and; if, furthermore, M is right divisor of every common left multiple of A and B, then M is a left least common multiple of A and B. Greatest common left divisor and the least common right multiple of two given matrices over commutative elementary divisor domain are uniquely determined up to invertible right factors.

Theorem 1. *Let* R *be a commutative elementary divisor domain* [1]. *Let an equation* $A = BX$, *where* $A, B \in M_n(R)$ *is solvable. Then the left greatest common divisor and the left least common multiple of its solutions are again its solution.*

Problem. Describe a rings in which the sets of the roots of the linear equations contain a generating elements.