the element $b_i(x) \neq 0$ *does not contain* n_i*-monomial,*

$$n_i = \begin{cases} 2co\deg a_i, \ i = 1, \ 3, \\ 2co\deg a_2' + k_1, \ i = 2, \end{cases},$$

$b_j(x) \equiv 0$. *The matrix* $B(x)$ *is uniquely defined.*

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Key words and phrases. Semiscalar equivalence of matrices, reduced matrix, canonical form

On greatest common divisors and least common multiple of linear matrix equation solutions

Volodymyr Shchedryk

Investigation of linear equation solutions has a profound history. Due to applied and theoretical problems we need to find roots with certain predefined properties. Matrix equations were studied with a symmetry condition, with Hermitian positively defined condition, with minimal rank condition on the solutions.

Let R be an associative ring with $1 \neq 0$. A set of all solutions of the equation $a = bx$ in R is $c + Ann_r(b)$, where c is some root one,

$$Ann_r(b) = \{f \in R | bf = 0\}.$$

Such a description of the roots is not always convenient. We would like to have their image in the form of a product. In this connection, the question arises search for the generating element of this set.

Let A, B be a matrices over ring R. If $A = BC$, then A is a right multiple of B and B is a left divisor of A. If $A = DA_1$ and $B = DB_1$, then D is a common left divisor of A, B; if, furthermore, D is a right multiple of every common right divisor of A and B, then D is a left greatest common divisor of A, B.

If $M = NA = KB$, then M is a common left multiple of A and B, and; if, furthermore, M is right divisor of every common left multiple of A and B, then M is a left least common multiple of A and B. Greatest common left divisor and the least common right multiple of two given matrices over commutative elementary divisor domain are uniquely determined up to invertible right factors.

Theorem 1. *Let* R *be a commutative elementary divisor domain* [1]. *Let an equation* $A = BX$, *where* $A, B \in M_n(R)$ *is solvable. Then the left greatest common divisor and the left least common multiple of its solutions are again its solution.*

Problem. Describe a rings in which the sets of the roots of the linear equations contain a generating elements.

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Key words and phrases. Commutative elementary divisor domain, matrix linear equation, greatest common left divisor, least common left multiple

Partition of Gaussian integers into a product of power-free numbers

Valeriia Shramko

We solve the problem of distribution of values of the function of the number of representations of Gaussian integers from a narrow sector in a product of power-free numbers.

Let G be a set of Gaussian integers. Let x be a growing to ∞ parameter. Let $S_\varphi(x)$ denote a sector of complex S-plane

$$S_\varphi(x) := \{\alpha \in G \mid \varphi_1 \leq arg\alpha \leq \varphi_2, N(\alpha) \leq x\}, \tag{1}$$

where $N(\alpha) =\mid \alpha \mid^2$.

Let $S_\varphi(x)$ be a narrow sector, if $\varphi_2 - \varphi_1 = o(x^{-\varepsilon})$ for $x \to \infty$, $\varepsilon > 0$ is a small positive integer.

A Gaussian integer α is power-free, if there is no Gaussian integer β such that $\alpha = \beta^k$, $k \in \{2, 3, \ldots\}$. Let us notice that all square-free numbers are power-free.

We have proved the following statements:

Theorem 1. *Let $g_2(\alpha)$ be the number of representations of a Gaussian integer α in the product of power-free numbers, where the positions of the factors are not count. For $x \to \infty$ the following asymptotic formula is true*

$$\sum_{N(\alpha)\leq x} g_2(\alpha) = x \sum_{n=0}^{\infty} d_n \frac{I_{n+1}(2\sqrt{\log x})}{(\log x)^{\frac{n+1}{2}}} + O(x), \tag{2}$$

where $I_n(x)$ is the modified Bessel's function of the first kind, coefficients d_n, $n \geq 1$, can be defined through coefficients from the decomposition of function $F(s)$ in a Taylor's series. The function $F(s)$ can be defined through an expression for the generating function of $g_2(\alpha)$

$$F_2(s) = \sum_{0\neq\alpha'\in G} \frac{g_2(\alpha)}{N^s(\alpha)} = exp\left(\frac{\pi}{s-1} + F(s)\right). \tag{3}$$

Theorem 2. *Let $g_2^*(\alpha)$ be the number of representations of a Gaussian integer α in the product $\alpha = \delta_1\delta_2\ldots\delta_k$, where δ_i, $i = \overline{1;k}$, are power-free numbers, $N(\beta_1) \leq N(\beta_2) \leq \ldots \leq$*