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Partition of Gaussian integers into a product of power-free numbers

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We solve the problem of distribution of values of the function of the number of representations of Gaussian integers from a narrow sector in a product of power-free numbers.

Let G be a set of Gaussian integers. Let x be a growing to ∞ parameter. Let $S_\varphi(x)$ denote a sector of complex S-plane

$$S_\varphi(x) := \{\alpha \in G \mid \varphi_1 \leq arg\alpha \leq \varphi_2, N(\alpha) \leq x\}, \tag{1}$$

where $N(\alpha) = \mid \alpha \mid^2$.

Let $S_\varphi(x)$ be a narrow sector, if $\varphi_2 - \varphi_1 = o(x^{-\varepsilon})$ for $x \to \infty$, $\varepsilon > 0$ is a small positive integer.

A Gaussian integer α is power-free, if there is no Gaussian integer β such that $\alpha = \beta^k$, $k \in \{2, 3, \ldots\}$. Let us notice that all square-free numbers are power-free.

We have proved the following statements:

Theorem 1. *Let $g_2(\alpha)$ be the number of representations of a Gaussian integer α in the product of power-free numbers, where the positions of the factors are not count. For $x \to \infty$ the following asymptotic formula is true*

$$\sum_{N(\alpha)\leq x} g_2(\alpha) = x \sum_{n=0}^{\infty} d_n \frac{I_{n+1}(2\sqrt{\log x})}{(\log x)^{\frac{n+1}{2}}} + O(x), \tag{2}$$

where $I_n(x)$ is the modified Bessel's function of the first kind, coefficients d_n, $n \geq 1$, can be defined through coefficients from the decomposition of function $F(s)$ in a Taylor's series. The function $F(s)$ can be defined through an expression for the generating function of $g_2(\alpha)$

$$F_2(s) = \sum_{0\neq\alpha'\in G} \frac{g_2(\alpha)}{N^s(\alpha)} = exp\left(\frac{\pi}{s-1} + F(s)\right). \tag{3}$$

Theorem 2. *Let $g_2^*(\alpha)$ be the number of representations of a Gaussian integer α in the product $\alpha = \delta_1\delta_2\ldots\delta_k$, where δ_i, $i = \overline{1;k}$, are power-free numbers, $N(\beta_1) \leq N(\beta_2) \leq \ldots \leq$*

$N(\beta_k)$. Then

$$\sum_{N(\alpha)\le x} g_2^*(\alpha) \sim e^{c_0\sqrt{\log x}} \sum_{(h,\upsilon)} H(h,\upsilon)(\log x)^{-\frac{2h+\upsilon}{4}} \left(1 + a_0(\log x)^{-\frac{1}{2}} - \frac{2h+\upsilon}{4}(\log x)^{-1}\right), \quad (4)$$

where c_0, a_0 are positive countable constants, the sum $\sum_{(h,\upsilon)}$ means that we summarize by all the pairs (h,υ) such that $1 \le h \le N$, $\upsilon = 1, 2, \ldots$ and $h + \frac{1}{2}\upsilon \le N + \frac{5}{2}$.

These results are a generalization of the results of K. Broughan [**1**] and I. Katai – M. V. Subbarao [**2**].

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The commutators of Sylow 2-subgroups of alternating group and wreath product. Their minimal generating sets

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We consider the commutator of Sylow 2-subgroups of an alternating group and research its minimal generating sets. The commutator width of a group G, denoted by $cw(G)$ [**1**], is the maximum of commutator lengths of elements of its derived subgroup $[G, G]$. The commutator width of Sylow 2-subgroups of the alternating group A_{2^k}, symmetric group S_{2^k} and $C_p \wr B$ are equal to 1. The paper presents a structure of a commutator subgroup of Sylow 2-subgroups of alternating groups. We prove that the commutator width [**1**] of an arbitrary element of a permutational wreath product of cyclic groups C_{p_i}, $p_i \in \mathbb{N}$, is 1. As it has been proven in [**2**] there are subgroups G_k and B_k in the automorphisms group $AutX^{[k]}$ of the restricted binary rooted tree such that $G_k \simeq Syl_2 A_{2^k}$ and $B_k \simeq Syl_2 S_{2^k}$, respectively.

Theorem 1. *An element $(g_1, g_2)\sigma \in G_k'$, where $\sigma \in S_2$ iff $g_1, g_2 \in G_{k-1}$ and $g_1 g_2 \in B_{k-1}'$.*

Lemma 1. *For any group B and integer $p \ge 2$ the following inequality is true:*

$$cw(B \wr C_p) \le \max(1, cw(B)).$$

Corollary 1. *For prime $p > 2$ and $k > 1$ the commutator widths of $Syl_p(A_{p^k})$ and of $Syl_p(S_{p^k})$ are equal to 1.*

Further, we analyze the structure of the elements of $Syl_2 S_{2^k}'$ and obtain the following result.

Theorem 2. *Elements of $Syl_2 S_{2^k}'$ have the following form*
$\{[f, l] \mid f \in B_k, l \in G_k\} = \{[l, f] \mid f \in B_k, l \in G_k\}$.