$N(\beta_k)$. Then

$$\sum_{N(\alpha)\le x} g_2^*(\alpha) \sim e^{c_0\sqrt{\log x}} \sum_{(h,\upsilon)} H(h,\upsilon)(\log x)^{-\frac{2h+\upsilon}{4}} \left(1 + a_0(\log x)^{-\frac{1}{2}} - \frac{2h+\upsilon}{4}(\log x)^{-1}\right), \quad (4)$$

where c_0, a_0 are positive countable constants, the sum $\sum_{(h,\upsilon)}$ means that we summarize by all the pairs (h,υ) such that $1 \le h \le N$, $\upsilon = 1, 2, \ldots$ and $h + \frac{1}{2}\upsilon \le N + \frac{5}{2}$.

These results are a generalization of the results of K. Broughan [**1**] and I. Katai – M. V. Subbarao [**2**].

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Key words and phrases. Gaussian integer, power-free number, square-free number

The commutators of Sylow 2-subgroups of alternating group and wreath product. Their minimal generating sets

Ruslan Skuratovskii

We consider the commutator of Sylow 2-subgroups of an alternating group and research its minimal generating sets. The commutator width of a group G, denoted by $cw(G)$ [**1**], is the maximum of commutator lengths of elements of its derived subgroup $[G,G]$. The commutator width of Sylow 2-subgroups of the alternating group A_{2^k}, symmetric group S_{2^k} and $C_p \wr B$ are equal to 1. The paper presents a structure of a commutator subgroup of Sylow 2-subgroups of alternating groups. We prove that the commutator width [**1**] of an arbitrary element of a permutational wreath product of cyclic groups C_{p_i}, $p_i \in \mathbb{N}$, is 1. As it has been proven in [**2**] there are subgroups G_k and B_k in the automorphisms group $AutX^{[k]}$ of the restricted binary rooted tree such that $G_k \simeq Syl_2 A_{2^k}$ and $B_k \simeq Syl_2 S_{2^k}$, respectively.

Theorem 1. *An element $(g_1, g_2)\sigma \in G_k'$, where $\sigma \in S_2$ iff $g_1, g_2 \in G_{k-1}$ and $g_1 g_2 \in B_{k-1}'$.*

Lemma 1. *For any group B and integer $p \ge 2$ the following inequality is true:*

$$cw(B \wr C_p) \le \max(1, cw(B)).$$

Corollary 1. *For prime $p > 2$ and $k > 1$ the commutator widths of $Syl_p(A_{p^k})$ and of $Syl_p(S_{p^k})$ are equal to 1.*

Further, we analyze the structure of the elements of $Syl_2 S_{2^k}'$ and obtain the following result.

Theorem 2. *Elements of $Syl_2 S_{2^k}'$ have the following form*
$\{[f,l] \mid f \in B_k, l \in G_k\} = \{[l,f] \mid f \in B_k, l \in G_k\}$.

Moreover, we get a more general result about the commutator width for a finite wreath product of finite cyclic groups.

Corollary 2. *If $W = C_{p_k} \wr \ldots \wr C_{p_1}$ then for $k \geq 2$ we have $cw(W) = 1$.*

Theorem 3. *The commutator width of the group $Syl_2 A_{2^k}$ is equal to 1 for $k \geq 2$.*

Theorem 4. *A commutator of G_k has the form $G'_k \simeq G_{k-1} \star G_{k-1}$, where $\star$ is the subdirect product. The order of G'_k is equal to 2^{2^k-k-2}. The order of G''_k is equal to 2^{2^k-3k+1}.*

Proposition 1. *The subgroup $(Syl_2 A_{2^k})'$ has a minimal generating set of $2k-3$ generators.*

For instance, a minimal generating set of $Syl'_2(A_8)$ consists of 3 generators: $(1,3)(2,4)(5,7)(6,8)$, $(1,2)(3,4)$, $(1,3)(2,4)(5,8)(6,7)$. In addition, $Syl'_2(A_8) \simeq C_2^3$.

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Key words and phrases. Commutator width, minimal generating set, commutator of Sylow 2-subgroups of alternating group, commutator width of wreath product

On general solutions of generalized ternary quadratic invertible functional equations of length three

Fedir Sokhatsky

A binary quasigroup is a pair $(Q;\circ)$, where Q is a set called a *carrier* and $\circ$ is an invertible binary operation defined on Q, i.e., there exist operations $\overset{\ell}{\circ}$ and $\overset{r}{\circ}$ such that for any x, $y \in Q$

$$(x \overset{\ell}{\circ} y) \circ y = x, \qquad (x \circ y) \overset{\ell}{\circ} y = x, \qquad x \circ (x \overset{r}{\circ} y) = y, \qquad x \overset{r}{\circ} (x \circ y) = y$$

are true. Similarly, a mapping $f\colon\ Q^3 \to Q$ is a *ternary invertible operation* if there exist operations ${}^{(14)}f$, ${}^{(24)}f$, ${}^{(34)}f$ such that for all x, y, z in Q

$$f({}^{(14)}f(x,y,z),y,z) = x, \qquad {}^{(14)}f(f(x,y,z),y,z) = x,$$
$$f(x,{}^{(24)}f(x,y,z),z) = y, \qquad {}^{(24)}f(x,f(x,y,z),z) = y,$$
$$f(x,y,{}^{(34)}f(x,y,z)) = z, \qquad {}^{(34)}f(x,y,f(x,y,z)) = z$$

hold. If an operation f is invertible, then the algebra $(Q; f, {}^{(14)}f, {}^{(24)}f, {}^{(34)}f)$ is called a *ternary quasigroup.*

Here, a *ternary functional equation* [**1, 2**] is a universally quantified equality $T_1 = T_2$, where T_1 and T_2 are terms consisting of individual and ternary functional variables, in addition all functional variables are free. The number of the functional variables including their repetitions is called a *length* of the equation. An equation is called generalized if all functional variables are pairwise different.