Canonical decompositions of solutions of the functional equations of generalized associativity are found in [**2**].

References

1. Aczél J., V. D. Belousov , Hosszú M. *Generalized associativity and bisymmetry on quasigroups* // Acta Math. Acad. Sci. Hungar. — 1960. — Vol. 11, Iss. 1-2. — P. 127–136.
2. F. M.Sokhatsky , H. V. Krainichuk *Solution of distributive-like quasigroup functional equations* // Comment. Math. Univ. Carolin. — 2012. — Vol. 53, 3. — P. 447–459.

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About group isotopes with inverse property

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A *quasigroup* is an algebra $(Q;\cdot;\overset{\ell}{\cdot};\overset{r}{\cdot})$ with identities

$$(x\cdot y)\overset{\ell}{\cdot}y=x,\qquad (x\overset{\ell}{\cdot}y)\cdot y=x,\qquad x\overset{r}{\cdot}(x\cdot y)=y,\qquad x\cdot(x\overset{r}{\cdot}y)=y.$$

They say that the operation $(\cdot)$ have: *left* (*right*, *middle*) inverse property [**1, 4**], if

$$\lambda x\cdot xy=y\qquad (\text{respectively},\quad yx\cdot\rho x=y,\quad x\cdot y=\mu(y\cdot x))$$

for some transformation λ, (resp. ρ, μ) of the set Q.

If the operation $(\cdot)$ in a quasigroup $(Q;\cdot;\overset{\ell}{\cdot};\overset{r}{\cdot})$ has a middle inverse property, then the operations $(\overset{\ell}{\cdot})$ and $(\overset{r}{\cdot})$ have left and right inverse property respectively.

Let $(Q;\circ)$ be a group isotope (i.e. it is isotopic to a group) and let $0\in Q$, then

$$x\circ y=\alpha x+a+\beta y \tag{1}$$

is called a *0-canonical decomposition*, if $(Q;+;0)$ is a group and $\alpha 0=\beta 0=0$. An arbitrary element of a group isotope uniquely defines its canonical decomposition [**2**].

Theorem 1. *Let $(Q;\circ)$ be a group isotope and (1) be its canonical decomposition, then:*

1) *$(\circ)$ has a right inverse property if and only if α an involutive automorphism of $(Q;+)$ and*

$$\alpha a=-a,\qquad \rho=\beta^{-1}JI_a\alpha\beta.$$

2) *$(\circ)$ has a left inverse property with if and only if β an involutive anti-automorphism of $(Q;+)$ and*

$$\beta a=-a,\qquad \lambda=\alpha^{-1}JI_a\beta\alpha,$$

3) *$(\circ)$ is middle inverse property if and only if exist anti-automorphism θ such that*

$$\mu x=\theta x+c,\qquad \theta^2=I_c^{-1},\qquad \alpha=\theta\beta,$$

where $c:=-\theta a+a$.

Let $\mathbb{Z}_{15}$ be a ring modulo 15, operations $(*)$, $(\circ)$, $(\bullet)$ defined on $\mathbb{Z}_{15}$ by the equalities

$$x * y := 2x + 3 + 4y, \qquad x \circ y := 4x + 3 + 2y, \qquad x \bullet y := 8x + 6 - 2y$$

have left, right and middle inverse properties respectively and $\lambda(x) = 11x$, $\rho(x) = 11x$, $\mu(x) = 11x$.

References

1. V.D. Belousov, *Foundations of the theory of quasigroups and loops*, (in Russian). Moscow, Nauka, 1967.
2. F.M. Sokhatsky, *On group isotopes II*, Ukrainian Math.J. 47(12) (1995), 1935–1948.
3. F.M. Sokhatsky, *Parastrophic symmetry in quasigroup theory*, Bulletin of Donetsk National University. Series A: Natural Sciences. (2016), no. 1/2, 70–83.
4. F. Sokhatsky, A. Lutsenko, *A truss of varieties of IP-quasigroups*. In: Abstracts of the young disciplines "Pidstryhach reading - 2019" Pidstryhach Institute for Applied Problems of Mechanics and Mathematics (in Ukrainian). Lviv 27-29 May, 2019. http://www.iapmm.lviv.ua/chyt2019/abstracts/Lucenko.pdf

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An invertibility criterion of composition of two multiary central quasigroups

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An n-ary operation f defined on a set Q is said to be *invertible* if it is invertible in each of the monoids $(\mathcal{O}_n, \underset{i}{\oplus})$ of all n-ary operations defined on Q, where

$$(f \underset{i}{\oplus} g)(x_0, \dots, x_{n-1}) := f(x_0, \dots, x_{i-1}, g(x_0, \dots, x_{n-1}), x_{i+1}, \dots, x_{n-1}), \quad i = 0, \dots, n-1.$$

An n-ary groupoid $(Q; f)$ is called: a *quasigroup*, if the operation is invertible and a *group isotope*, if there exists a group $(G; +)$ and bijections $\gamma_0, \dots, \gamma_n$ from Q to G such that

$$f(x_0, \dots, x_{n-1}) = \gamma_n^{-1}(\gamma_0 x_0 + \dots + \gamma_{n-1} x_{n-1})$$

for all $x_0, \dots x_{n-1}$ in Q. It is easy to verify that a group isotope is a quasigroup. Let 0 be an arbitrary element from Q, a sequence $(+, \alpha_0, \dots, \alpha_{n-1}, a)$ is said to be a *canonical decomposition* (see [**1**]) of a group isotope $(Q; f)$ if $(Q; +, 0)$ is a group, $\alpha_0 0 = \dots = \alpha_{n-1} 0 = 0$, $a \in Q$ and

$$f(x_0, \dots, x_{n-1}) = \alpha_0 x_0 + \dots + \alpha_{n-1} x_{n-1} + a.$$

$(Q; +, 0)$ is called a *canonical decomposition group* and $\alpha_0, \dots, \alpha_{n-1}$ are *coefficients*.

A group isotope is called *central*, if in a canonical decomposition the group is commutative and all coefficients are automorphisms of the group. A map $\alpha : A \to B$ is called *ortho-complete*,