Let $\mathbb{Z}_{15}$ be a ring modulo 15, operations $(*)$, $(\circ)$, $(\bullet)$ defined on $\mathbb{Z}_{15}$ by the equalities

$$x * y := 2x + 3 + 4y, \qquad x \circ y := 4x + 3 + 2y, \qquad x \bullet y := 8x + 6 - 2y$$

have left, right and middle inverse properties respectively and $\lambda(x) = 11x$, $\rho(x) = 11x$, $\mu(x) = 11x$.

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Key words and phrases. Quasigroup, IP-quasigroup, invertible function, group isotope

An invertibility criterion of composition of two multiary central quasigroups

Fedir Sokhatsky, Viktor Savchuk

An n-ary operation f defined on a set Q is said to be *invertible* if it is invertible in each of the monoids $(\mathcal{O}_n, \underset{i}{\oplus})$ of all n-ary operations defined on Q, where

$$(f \underset{i}{\oplus} g)(x_0, \dots, x_{n-1}) := f(x_0, \dots, x_{i-1}, g(x_0, \dots, x_{n-1}), x_{i+1}, \dots, x_{n-1}), \quad i = 0, \dots, n-1.$$

An n-ary groupoid $(Q; f)$ is called: a *quasigroup*, if the operation is invertible and a *group isotope*, if there exists a group $(G; +)$ and bijections $\gamma_0, \dots, \gamma_n$ from Q to G such that

$$f(x_0, \dots, x_{n-1}) = \gamma_n^{-1}(\gamma_0 x_0 + \dots + \gamma_{n-1} x_{n-1})$$

for all $x_0, \dots x_{n-1}$ in Q. It is easy to verify that a group isotope is a quasigroup. Let 0 be an arbitrary element from Q, a sequence $(+, \alpha_0, \dots, \alpha_{n-1}, a)$ is said to be a *canonical decomposition* (see [**1**]) of a group isotope $(Q; f)$ if $(Q; +, 0)$ is a group, $\alpha_0 0 = \dots = \alpha_{n-1} 0 = 0$, $a \in Q$ and

$$f(x_0, \dots, x_{n-1}) = \alpha_0 x_0 + \dots + \alpha_{n-1} x_{n-1} + a.$$

$(Q; +, 0)$ is called a *canonical decomposition group* and $\alpha_0, \dots, \alpha_{n-1}$ are *coefficients*.

A group isotope is called *central*, if in a canonical decomposition the group is commutative and all coefficients are automorphisms of the group. A map $\alpha : A \to B$ is called *ortho-complete*, if all preimages of elements from the set B have the same cardinality. Thus, an n-ary operation f defined on an m-element set Q is ortho-complete if for any $a \in Q$ the equation $f(x_1, \ldots, x_n) = a$ has exactly m^{n-1} solutions.

Lemma 1. [**2**] *A finite n-ary quasigroup $(Q; f)$ is admissible iff for some $k \in \overline{1,n} := \{1, 2, \ldots, n\}$ there exists an $(n-1)$-ary invertible operation g such that the operation h defined by*

$$h(x_1, \ldots, x_{k-1}, x_{k+1}, \ldots, x_n) = f(x_1, \ldots, x_{k-1}, g(x_1, \ldots, x_{k-1}, x_{k+1}, \ldots, x_n), x_{k+1}, \ldots, x_n) \quad (1)$$

is ortho-complete.

Theorem 1. *Let $(+, \alpha_0, \ldots, \alpha_{n-1}, a)$ and $(+, \beta_1, \ldots, \beta_{k-1}, \beta_{k+1}, \ldots, \beta_n, b)$ be canonical decompositions of central quasigroups $(Q; f)$ and $(Q; g)$ respectively. Then $(n-1)$-ary operation h defined by (1) is invertible iff for all $i \in \overline{1,n} \setminus \{k\}$ the endomorphism $\alpha_i + \alpha_k\beta_i$ of the group $(Q; +, 0)$ is its automorphism.*

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Key words and phrases. Quasigroup, linear quasigroup, automorphism, canonical decomposition

On total multiplication groups

Parascovia Syrbu

The concept of multiplication group was introduced by Albert in the middle of the 20th century and at present is a standard tool in the algebraic quasigroup (loop) theory. Let $(Q, \cdot)$ be a quasigroup. The left, right and middle translations are denoted by L_x, R_x, J_x, respectively, and are defined as follows: $L_x(y) = x \cdot y$, $R_x(y) = y \cdot x$, $J_x(y) = y \backslash x$, $\forall x, y \in Q$. The groups $Mlt(Q) =< L_x, R_x | x \in Q >$ and $TMlt(Q) =< L_x, R_x, J_x | x \in Q >$ are called the multiplication group and the total multiplication group of $(Q, \cdot)$, respectively. If $(Q, \cdot)$ is a loop, then the stabilizer of its unit in $Mlt(Q)$ (resp. in $TMlt(Q)$) is called the inner mapping group (resp. the total inner mapping group) of $(Q, \cdot)$ and is denoted by $Inn(Q)$ (resp. $TInn(Q)$).

The total multiplication groups and total inner mapping groups have been considered at the end of 60s by Belousov in [**1**], where he noted that $TMlt(Q)$ is invariant under the parastrophy of quasigroups and gave a set of generators for the group $TInn(Q)$. Sets of generators of the total inner mapping group of a loop are also given by D. Stanovsky, P. Vojtechovsky [**2**], V. Shcherbacov [**4**] and P. Syrbu [**3**]. It is known that the total multiplication groups of isostrophic