if all preimages of elements from the set B have the same cardinality. Thus, an n-ary operation f defined on an m-element set Q is ortho-complete if for any $a \in Q$ the equation $f(x_1, \ldots, x_n) = a$ has exactly m^{n-1} solutions.

Lemma 1. **[2]** *A finite n-ary quasigroup $(Q; f)$ is admissible iff for some $k \in \overline{1,n} := \{1, 2, \ldots, n\}$ there exists an $(n-1)$-ary invertible operation g such that the operation h defined by*

$$h(x_1, \ldots, x_{k-1}, x_{k+1}, \ldots, x_n) = f(x_1, \ldots, x_{k-1}, g(x_1, \ldots, x_{k-1}, x_{k+1}, \ldots, x_n), x_{k+1}, \ldots, x_n) \quad (1)$$

is ortho-complete.

Theorem 1. *Let $(+, \alpha_0, \ldots, \alpha_{n-1}, a)$ and $(+, \beta_1, \ldots, \beta_{k-1}, \beta_{k+1}, \ldots, \beta_n, b)$ be canonical decompositions of central quasigroups $(Q; f)$ and $(Q; g)$ respectively. Then $(n-1)$-ary operation h defined by (1) is invertible iff for all $i \in \overline{1,n} \setminus \{k\}$ the endomorphism $\alpha_i + \alpha_k\beta_i$ of the group $(Q; +, 0)$ is its automorphism.*

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On total multiplication groups

Parascovia Syrbu

The concept of multiplication group was introduced by Albert in the middle of the 20th century and at present is a standard tool in the algebraic quasigroup (loop) theory. Let $(Q, \cdot)$ be a quasigroup. The left, right and middle translations are denoted by L_x, R_x, J_x, respectively, and are defined as follows: $L_x(y) = x \cdot y$, $R_x(y) = y \cdot x$, $J_x(y) = y \backslash x$, $\forall x, y \in Q$. The groups $Mlt(Q) =< L_x, R_x | x \in Q >$ and $TMlt(Q) =< L_x, R_x, J_x | x \in Q >$ are called the multiplication group and the total multiplication group of $(Q, \cdot)$, respectively. If $(Q, \cdot)$ is a loop, then the stabilizer of its unit in $Mlt(Q)$ (resp. in $TMlt(Q)$) is called the inner mapping group (resp. the total inner mapping group) of $(Q, \cdot)$ and is denoted by $Inn(Q)$ (resp. $TInn(Q)$).

The total multiplication groups and total inner mapping groups have been considered at the end of 60s by Belousov in **[1]**, where he noted that $TMlt(Q)$ is invariant under the parastrophy of quasigroups and gave a set of generators for the group $TInn(Q)$. Sets of generators of the total inner mapping group of a loop are also given by D. Stanovsky, P. Vojtechovsky **[2]**, V. Shcherbacov **[4]** and P. Syrbu **[3]**. It is known that the total multiplication groups of isostrophic loops are isomorphic. Also, it is known that the multiplication groups of IP-loops are normal subgroups of index two of the total multiplication groups.

We consider general properties of the total multiplication groups of loops and the behavior of the multiplication group in the total multiplication group. Some classes of loops, for which the multiplication group (the inner mapping group) is a normal subgroup of the total multiplication group (resp. of its total inner mapping group) are found. Characterizations of the quotient groups $TMlt(Q)/Mlt(Q)$ and $TInn(Q)/Inn(Q)$ are given, where Q is a middle Bol loop.

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General solutions of generalized ternary quadratic quasigroup functional equations of length three

Alla Tarasevych

Let Q be a set. A mapping $f:\ Q^3 \to Q$ is called a *ternary invertible operation* if there exist operations ${}^{(14)}f$, ${}^{(24)}f$, ${}^{(34)}f$ such that for any $x, y, z \in Q$ the following identities

$$f({}^{(14)}f(x,y,z),y,z) = x, \qquad {}^{(14)}f(f(x,y,z),y,z) = x,$$

$$f(x,{}^{(24)}f(x,y,z),z) = y, \qquad {}^{(24)}f(x,f(x,y,z),z) = y,$$

$$f(x,y,{}^{(34)}f(x,y,z)) = z, \qquad {}^{(34)}f(x,y,f(x,y,z)) = z$$

hold [**2**]. The algebra $(Q; f, {}^{(14)}f, {}^{(24)}f, {}^{(34)}f)$ is a *ternary quasigroup* if it satisfies the above identities. An operation f is called a *left-universally neutral* if $f(x,y,y) = x$.

A universally quantified equality $T_1 = T_2$ is a *ternary functional equation* if the terms T_1, T_2 consist of individual and ternary functional variables, in addition the functional variables are free [**1, 2**]. The ternary functional equation is *generalized* if all functional variables are pairwise different; *quadratic* if each individual variable has exactly two appearances. The number of all functional variables including their repetitions is *length* of the equation. Let (F_1, F_2, F_3) be the lexicographic sequence of all different functional variables of the equation $T_1 = T_2$, then a triplet (f_1, f_2, f_3) of functions defined on the set Q is called a *solution on* Q, if the proposition obtained from $T_1 = T_2$ by replacing F_1 with f_1, F_2 with f_2, F_3 with f_3 is true [**2**].

A full classification of generalized ternary quadratic functional equations of length three up to parastrophically primary equivalence is given in [**3**]. There are exactly four pairwise non-equivalent equations. The set of all quasigroup solutions of one of them is described in [**4**], and such sets of the remaining three equations are found in the following theorems.