loops are isomorphic. Also, it is known that the multiplication groups of $IP$-loops are normal subgroups of index two of the total multiplication groups.

We consider general properties of the total multiplication groups of loops and the behavior of the multiplication group in the total multiplication group. Some classes of loops, for which the multiplication group (the inner mapping group) is a normal subgroup of the total multiplication group (resp. of its total inner mapping group) are found. Characterizations of the quotient groups $TMlt(Q)/Mlt(Q)$ and $TInn(Q)/Inn(Q)$ are given, where $Q$ is a middle Bol loop.

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Contact information


**Parascovia Syrbu**
Department of Mathematics, University of Moldova, Chisinau, Republic of Moldova
*Email address*: syrbuviv@yahoo.com


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# General solutions of generalized ternary quadratic quasigroup functional equations of length three

Alla Tarasevych

Let $Q$ be a set. A mapping $f:\ Q^3 \to Q$ is called a *ternary invertible operation* if there exist operations ${}^{(14)}f$, ${}^{(24)}f$, ${}^{(34)}f$ such that for any $x, y, z \in Q$ the following identities

$$f({}^{(14)}f(x,y,z),y,z) = x, \qquad {}^{(14)}f(f(x,y,z),y,z) = x,$$

$$f(x,{}^{(24)}f(x,y,z),z) = y, \qquad {}^{(24)}f(x,f(x,y,z),z) = y,$$

$$f(x,y,{}^{(34)}f(x,y,z)) = z, \qquad {}^{(34)}f(x,y,f(x,y,z)) = z$$

hold [**2**]. The algebra $(Q; f, {}^{(14)}f, {}^{(24)}f, {}^{(34)}f)$ is a *ternary quasigroup* if it satisfies the above identities. An operation $f$ is called a *left-universally neutral* if $f(x,y,y) = x$.

A universally quantified equality $T_1 = T_2$ is a *ternary functional equation* if the terms $T_1$, $T_2$ consist of individual and ternary functional variables, in addition the functional variables are free [**1, 2**]. The ternary functional equation is *generalized* if all functional variables are pairwise different; *quadratic* if each individual variable has exactly two appearances. The number of all functional variables including their repetitions is *length* of the equation. Let $(F_1, F_2, F_3)$ be the lexicographic sequence of all different functional variables of the equation $T_1 = T_2$, then a triplet $(f_1, f_2, f_3)$ of functions defined on the set $Q$ is called a *solution on $Q$*, if the proposition obtained from $T_1 = T_2$ by replacing $F_1$ with $f_1$, $F_2$ with $f_2$, $F_3$ with $f_3$ is true [**2**].

A full classification of generalized ternary quadratic functional equations of length three up to parastrophically primary equivalence is given in [**3**]. There are exactly four pairwise non-equivalent equations. The set of all quasigroup solutions of one of them is described in [**4**], and such sets of the remaining three equations are found in the following theorems.

THEOREM 1. *A triplet $(f_1, f_2, f_3)$ of ternary invertible operations is a solution of the equation $F_1(z, x, F_2(x, y, y)) = F_3(z, u, u)$ if and only if there exist left-universally neutral invertible operations $h_1$, $h_2$, $h_3$ and bijections $\alpha$, $\beta$ such that*

$$f_1(x, y, z) = h_1(\alpha x, y, \beta^{-1}z), \qquad f_2(x, y, z) = h_2(\beta x, y, z), \qquad f_3(x, y, z) = h_3(\alpha x, y, z).$$

THEOREM 2. *A triplet $(f_1, f_2, f_3)$ of ternary invertible operations is a solution of the equation $F_1(F_2(x, y, y), z, z) = F_3(x, u, u)$ if and only if there exist left-universally neutral invertible operations $g_1$, $g_2$, $g_3$ and bijections $\gamma$, $\delta$ such that*

$$f_1(x, y, z) = g_1(\gamma x, y, z), \qquad f_2(x, y, z) = g_2(\delta x, y, z), \qquad f_3(x, y, z) = g_3(\gamma\delta x, y, z).$$

THEOREM 3. *A triplet $(f_1, f_2, f_3)$ of ternary operations defined on a set $Q$ is a quasigroup solution of the functional equation $F_1(F_2(x, y, z), u, u) = F_3(x, y, z)$ if and only if the operation $f_2$ is invertible, there exist a bijection $\mu$ and a left-universally neutral operation $g$ such that*

$$f_3(x, y, z) = \mu f_2(x, y, z), \qquad f_1(x, y, z) = g(\mu x, y, z).$$

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CONTACT INFORMATION


**Alla Tarasevych**
Department of Higher Mathematics and Computer Science, Khmelnytskyi National University, Khmelnytskyi, Ukraine
*Email address*: allatarasevych@gmail.com


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# Classification of generalized ternary quadratic quasigroup functional equations of length three

ALLA TARASEVYCH, FEDIR SOKHATSKY

Let $Q$ be an arbitrary fixed set called a *carrier*. A mapping $f\colon\ Q^3 \to Q$ is called a *ternary invertible function*, if there exist functions ${}^{(14)}f, {}^{(24)}f, {}^{(34)}f$ such that for any $x, y, z \in Q$ the following identities:

$$\begin{aligned} &f({}^{(14)}f(x, y, z), y, z) = x, &&{}^{(14)}f(f(x, y, z), y, z) = x,\\ &f(x, {}^{(24)}f(x, y, z), z) = y, &&{}^{(24)}f(x, f(x, y, z), z) = y, \qquad (1)\\ &f(x, y, {}^{(34)}f(x, y, z)) = z, &&{}^{(34)}f(x, y, f(x, y, z)) = z \end{aligned}$$

hold. If an operation $f$ is invertible, then the algebra $(Q; f, {}^{(14)}f, {}^{(24)}f, {}^{(34)}f)$ is called a *ternary quasigroup* [**3**].