Theorem 1. *A triplet* (f_1, f_2, f_3) *of ternary invertible operations is a solution of the equation* $F_1(z, x, F_2(x, y, y)) = F_3(z, u, u)$ *if and only if there exist left-universally neutral invertible operations* h_1, h_2, h_3 *and bijections* α, β *such that*

$$f_1(x, y, z) = h_1(\alpha x, y, \beta^{-1} z), \qquad f_2(x, y, z) = h_2(\beta x, y, z), \qquad f_3(x, y, z) = h_3(\alpha x, y, z).$$

Theorem 2. *A triplet* (f_1, f_2, f_3) *of ternary invertible operations is a solution of the equation* $F_1(F_2(x, y, y), z, z) = F_3(x, u, u)$ *if and only if there exist left-universally neutral invertible operations* g_1, g_2, g_3 *and bijections* γ, δ *such that*

$$f_1(x, y, z) = g_1(\gamma x, y, z), \qquad f_2(x, y, z) = g_2(\delta x, y, z), \qquad f_3(x, y, z) = g_3(\gamma\delta x, y, z).$$

Theorem 3. *A triplet* (f_1, f_2, f_3) *of ternary operations defined on a set* Q *is a quasigroup solution of the functional equation* $F_1(F_2(x, y, z), u, u) = F_3(x, y, z)$ *if and only if the operation* f_2 *is invertible, there exist a bijection* μ *and a left-universally neutral operation* g *such that*

$$f_3(x, y, z) = \mu f_2(x, y, z), \qquad f_1(x, y, z) = g(\mu x, y, z).$$

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Key words and phrases. Ternary quasigroup, quadratic equation, length of a functional equation, parastrophically primary equivalence

Classification of generalized ternary quadratic quasigroup functional equations of length three

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Let Q be an arbitrary fixed set called a *carrier*. A mapping $f\colon Q^3 \to Q$ is called a *ternary invertible function*, if there exist functions ${}^{(14)}f, {}^{(24)}f, {}^{(34)}f$ such that for any $x, y, z \in Q$ the following identities:

$$\begin{aligned} &f({}^{(14)}f(x, y, z), y, z) = x, &&{}^{(14)}f(f(x, y, z), y, z) = x,\\ &f(x, {}^{(24)}f(x, y, z), z) = y, &&{}^{(24)}f(x, f(x, y, z), z) = y, \qquad (1)\\ &f(x, y, {}^{(34)}f(x, y, z)) = z, &&{}^{(34)}f(x, y, f(x, y, z)) = z \end{aligned}$$

hold. If an operation f is invertible, then the algebra $(Q; f, {}^{(14)}f, {}^{(24)}f, {}^{(34)}f)$ is called a *ternary quasigroup* [**3**].

Let Δ_3 be the set of all invertible ternary functions defined on a carrier Q. The formulas (1) are true for all invertible ternary functions that is to say, the above *hyperidentities* are true over the set Δ_3.

A universally quantified equality $T_1 = T_2$ is a *ternary functional equation* where T_1 and T_2 are terms consisting of individual and ternary functional variables, in addition all functional variables are free [**1, 3**]. We consider only generalized ternary quadratic quasigroup functional equations of length three, where the notion 'ternary quasigroup equation' means that all functional variables take their values only in the set of ternary invertible functions; the word 'generalized' means that the variables are pairwise different; the word 'quadratic' means that every individual variable has exactly two appearances or none; the notion 'length of a functional equation' is the number of functional variables including their repetitions.

Definition. ([**2**]) Two functional equations are called parastrophically primarily equivalent if one can be obtained from the other in a finite number of the following steps: 1) replacing the equation sides; 2) renaming the functional variables; 3) renaming the individual variables; 4) applying the hyperidentities (1).

Theorem. *Every generalized quadratic ternary quasigroup functional equation of length three is parastrophically primarily equivalent to exactly one of the following equations:*

$$F_1(z, x, F_2(x, y, y)) = F_3(z, u, u), \qquad F_1(F_2(x, y, z), u, u) = F_3(x, y, z),$$
$$F_1(F_2(x, y, y), z, z) = F_3(x, u, u), \qquad F_1(F_2(x, y, z), x, u) = F_3(y, z, u).$$

The general solutions of each of these functional equations have been found in [**4, 5**].

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Key words and phrases. Ternary quasigroup, quadratic equation, length of a functional equation, parastrophically primarily equivalent