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Classes of finite rings

Aleksandr Tsarev

The concept of formation appeared first in the 1960s in connection with finite solvable groups [**1**]. Further research showed that formations are of general algebraic nature and can be applied to the study of not necessarily solvable finite and infinite groups, Lie algebras, universal algebras and even of a general algebraic system [**2**]. A well-known result in group theory states that any formation of finite groups is saturated iff it is local (see Theorem 4.6 in the book [**3**]). In contrast to the group case, not every saturated formation of Lie and Leibniz algebras, monoids, rings, etc. can be locally defined. However, these formations have found various applications.

Commutative rings have found some interesting applications in cryptography and coding theory. Wedderburn's little theorem says every finite domain is a field. Codes over finite filds, which appear in information and communication theory, have been investigated as images of codes over Galois rings over the ring of integers modulo 4; see [**4**, Chapter 8]. These facts motivate to study properties of finite rings and their classes. In particular it will be interesting to translate some of the most powerful results of the theory of formations of finite groups for formations of rings. A class of rings is a *formation* whenever it contains all homomorphic images of its members and if it is subdirect product closed.

Theorem 1. [**5**] *The lattice of all formations of finite rings is algebraic and modular.*

Fuzzy sets became applied in fields such as pattern recognition, machine learning and data mining [**6**]. Focusing on the structure of ring, Liu [**7**] introduced and studied the notions of fuzzy subrings and fuzzy ideals, and showed that the images and preimages under onto homomorphisms of fuzzy ideals are fuzzy ideals, but not all the results on rings can be fuzzified.

Let R be a finite commutative ring with an identity element. It is established that there is a one-to-one correspondence between the set of all invariant fuzzy prime ideals of R and the set of all fuzzy prime ideals of each ring of the formation generated by R; see [**5**] for more details.

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On induced modules over group rings of groups of finite rank

Anatolii V. Tushev

Let G be a group and k be a field. A kG-module M is said to be imprimitive if there are a subgroup $H < G$ and a kH-submodule $N \leq M$ such that $M = N \otimes_{kH} KG$. If the module M is not imprimitive then it is said to be primitive. A representation of the group G is said to be primitive if the module of the representation is primitive.

Let G be a group of finite rank $r(G)$ and k be a field. A kG-module M is said to be semi-imprimitive if there are subgroup $H < G$ and a kH-submodule $N \leq M$ such that $r(H) < r(G)$ and $M = N \otimes_{kH} KG$. If the module M is not semi-imprimitive then it is said to be semi-primitive. A representation of the group G is said to be semi-primitive if the module of the representation is semi-primitive. An element $g \in G$ (a subgroup $H \leq G$) is said to be orbital if $|G : C_G(g)| < \infty$ ($|G : N_G(H)| < \infty$). The set Δ (G) of all orbital elements of G is a characteristic subgroup of G which is said to be the FC-center of G.

In [**1**] Harper shoved that any finitely generated not abelian-by-finite nilpotent group has an irreducible primitive representation over any not locally finite field. In [**3**] we proved that in the class of soluble groups of finite rank with the maximal condition for normal subgroups only polycyclic groups may have irreducible primitive faithful representations over a field of characteristic zero. In [**2**] Harper proved that if a polycyclic group G has a faithful irreducible semi-primitive representation then $A \bigcap \Delta$ (G) $\neq 1$ for any orbital subgroup A of G. It is well known that any polycyclic group is liner and has finite rank.

Theorem 1. *Let G be a linear group of finite rank. Suppose that G has a normal subgroup $1 \neq A$, such that $A \bigcap \Delta$ $(G) = 1$. Let k be a field of characteristic zero and let M be an irreducible kG-module such that $C_G(M) = 1$. Then there are a subgroup $S \leq G$ and a kS-submodule $U \leq M$ such that $r(S) < (G)$ and $M = U \otimes_{kS} kG$.*

Corollary 1. *Let G be a linear group of finite rank. If the group G has a faithful irreducible semi-primitive representation over a field of characteristic zero then $A \bigcap \Delta$ $(G) \neq 1$ for any orbital subgroup A of G.*

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