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On induced modules over group rings of groups of finite rank

Anatolii V. Tushev

Let G be a group and k be a field. A kG-module M is said to be imprimitive if there are a subgroup $H < G$ and a kH-submodule $N \leq M$ such that $M = N \otimes_{kH} KG$. If the module M is not imprimitive then it is said to be primitive. A representation of the group G is said to be primitive if the module of the representation is primitive.

Let G be a group of finite rank $r(G)$ and k be a field. A kG-module M is said to be semi-imprimitive if there are subgroup $H < G$ and a kH-submodule $N \leq M$ such that $r(H) < r(G)$ and $M = N \otimes_{kH} KG$. If the module M is not semi-imprimitive then it is said to be semi-primitive. A representation of the group G is said to be semi-primitive if the module of the representation is semi-primitive. An element $g \in G$ (a subgroup $H \leq G$) is said to be orbital if $|G : C_G(g)| < \infty$ ($|G : N_G(H)| < \infty$). The set Δ (G) of all orbital elements of G is a characteristic subgroup of G which is said to be the FC-center of G.

In [**1**] Harper shoved that any finitely generated not abelian-by-finite nilpotent group has an irreducible primitive representation over any not locally finite field. In [**3**] we proved that in the class of soluble groups of finite rank with the maximal condition for normal subgroups only polycyclic groups may have irreducible primitive faithful representations over a field of characteristic zero. In [**2**] Harper proved that if a polycyclic group G has a faithful irreducible semi-primitive representation then $A \bigcap \Delta$ (G) $\neq 1$ for any orbital subgroup A of G. It is well known that any polycyclic group is liner and has finite rank.

Theorem 1. *Let G be a linear group of finite rank. Suppose that G has a normal subgroup $1 \neq A$, such that $A \bigcap \Delta$ $(G) = 1$. Let k be a field of characteristic zero and let M be an irreducible kG-module such that $C_G(M) = 1$. Then there are a subgroup $S \leq G$ and a kS-submodule $U \leq M$ such that $r(S) < (G)$ and $M = U \otimes_{kS} kG$.*

Corollary 1. *Let G be a linear group of finite rank. If the group G has a faithful irreducible semi-primitive representation over a field of characteristic zero then $A \bigcap \Delta$ $(G) \neq 1$ for any orbital subgroup A of G.*

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On irreducibility of monomial matrices of order 7 over local rings

ALEXANDER TYLYSHCHAK

The problem of classifying, up to similarity, all the matrices over a commutative ring (which is not a field) is usually very difficult; in most cases it is “unsolvable” (wild, as in the case of the rings of residue classes considered by Bondarenko [**1**]). In such situation, an important place is occupied by irreducible and indecomposable matrices over rings.

Let R be a commutative local ring with identity with Jacobson radical $\operatorname{Rad} R = tR$, $t \neq 0$, n, k be a natural, $0 < k < n$,

$$M(t,k,n) = \begin{pmatrix} \overbrace{\begin{matrix} 0 & \dots & 0 \\ 1 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & 1 \\ 0 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & 0 \end{matrix}}^{k} & \begin{matrix} 0 & \dots & 0 & t \\ 0 & \dots & 0 & 0 \\ \vdots & \ddots & \vdots & \vdots \\ 0 & \dots & 0 & 0 \\ t & \dots & 0 & 0 \\ \vdots & \ddots & \vdots & \vdots \\ 0 & \dots & t & 0 \end{matrix} \end{pmatrix}$$

be an $n \times n$-matrix. This matrices first arose in studying indecomposable representations of finite p-groups over commutative local rings [**2**].

The question when matrix $M(t,k,n)$ is reducible had been solved, in particular, in following cases.

$M(t,k,n)$	Case	Sourse
irreducible	$k = 1, n-1,\quad t \neq 0$	[**2**]
reducible	$(k,n) > 1$	[**3**]
irreducible	$n < 7$, $(k,n) = 1,\quad t \neq 0$	[**4**]
reducible	$n = 7$, $k = 3, 4,\quad t^2 = 0$	[**4, 5**]

THEOREM 1. *Let $n = 7$, $0 < k < n$, $t^2 \neq 0$. The matrix $M(t,k,n)$ is irreducible over R.*

These studies were carried out together with V. M. Bondarenko.

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