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On irreducibility of monomial matrices of order 7 over local rings

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The problem of classifying, up to similarity, all the matrices over a commutative ring (which is not a field) is usually very difficult; in most cases it is “unsolvable” (wild, as in the case of the rings of residue classes considered by Bondarenko [**1**]). In such situation, an important place is occupied by irreducible and indecomposable matrices over rings.

Let R be a commutative local ring with identity with Jacobson radical $\operatorname{Rad} R = tR$, $t \neq 0$, n, k be a natural, $0 < k < n$,

$$M(t,k,n) = \begin{pmatrix} \overbrace{0 & \dots & 0}^{k} & 0 & \dots & 0 & t \\ 1 & \dots & 0 & 0 & \dots & 0 & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & \dots & 1 & 0 & \dots & 0 & 0 \\ 0 & \dots & 0 & t & \dots & 0 & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & \dots & 0 & 0 & \dots & t & 0 \end{pmatrix}$$

be an $n \times n$-matrix. This matrices first arose in studying indecomposable representations of finite p-groups over commutative local rings [**2**].

The question when matrix $M(t,k,n)$ is reducible had been solved, in particular, in following cases.

$M(t,k,n)$	Case	Sourse
irreducible	$k = 1, n-1,$ $\quad t \neq 0$	[**2**]
reducible	$(k,n) > 1$	[**3**]
irreducible	$n < 7$, $(k,n) = 1,$ $\quad t \neq 0$	[**4**]
reducible	$n = 7$, $k = 3, 4,$ $\quad t^2 = 0$	[**4, 5**]

THEOREM 1. *Let $n = 7$, $0 < k < n$, $t^2 \neq 0$. The matrix $M(t,k,n)$ is irreducible over R.*

These studies were carried out together with V. M. Bondarenko.

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Can one hear the shape of permutation group?

Vasyl Ustimenko

Missing definitions of the theory of permutation groups and algebraic graph theory such as coherent configuratuion, orbital of permutation group, distance regular and distance transitive graphs, extended bipartite double of graph reader can find in [1], [2] or [3]. Connection of concept of coherent configurations with other objects of algebraic combinatorics are given in [4]. Concepts of Spectral Graph Theory could be found in [5].

Let G be finite group acting faithfully on set Ω. Corresponding group coherent configuration $GCC(G,\Omega)$ is a totality of all invariant binary relations of (G,Ω), i.e subsets of $\Omega\times\Omega$ which are unions of orbitals (G,Ω) (orbits of natural action of G on the set $\Omega\times\Omega$). We say that two non-similar group permutations (G,Ω) and (H,Ω) are isospectral if for each simple graph of symmetric binary relation of $GCC(G,\Omega)$ there exists isospectral graph of $GCC(H,\Omega)$.

Question written in the title of the abstract is about the existence of isospectral pair of permutation groups. We will say that two isospectralpermutation groups (G,Ω) and (H,Ω) are nonisomorphic if group G and H are nonisomorphic. Let $X_n(q)$ be Shevalley group corresponding to Coxeter-Dynkin diagram X_n, $W(X_n)$ be the Coxeter system of Weyl group of root system X_n. Symbols $\Gamma(X_n(q))$ and $\Gamma(W(X_n))$ stand for corresponding Lie and Coxeter geometries. $\Gamma_i(X_n(q))$ and $\Gamma_i(W(X_n))$ are totalities of geometry elements of type corresponding to node i of the diagram. There is natural correspondence between binary relations of $GCC(X_n(q),\Gamma(X_n(q)))$ and $GCC(W(X_n),\Gamma(W(X_n))$. Symbol X stands here for one of characters A,B,C,D. Notice that $GCC(W(B_n),\Gamma(W(B_n)) = GCC(W(C_n),\Gamma(W(C_n))$. Let ϕ be binary relation of $GCC(W(X_n),\Gamma(W(X_n)))$ and $\phi(X,q)$ correspond to ϕ as binary relation in $\Gamma(X_n(q))\times\Gamma(X_n(q))$.

Theorem 1. *If q is odd and $n\geq 3$ then pairs $(B_n(q),\Gamma_n(q))$, $(C_n(q),\Gamma_n(q))$ form families of nonisomorphicisospectral permutation groups.*

Corollary 1. *Let ϕ_i be binary relation of $GCC(W(B_n),\Gamma(W(B_n))$ containing in $\Gamma_i(W(B_n))\times\Gamma_i(W(B_n))$ and $\phi_i(B_n,q)$ and $\phi_i(C_n,q)$are corresponding to ϕ_i invariant binary relationsof $B_n(q)$ and $C_n(q)$ containing in $\Gamma_i(B_n(q))\times\Gamma_i(B_n(q))$ and $\Gamma_i(C_n(q))\times\Gamma_i(C_n(q))$. If q is odd then graphs $\phi_i(B_n,q)$ and $\phi_i(C_n,q)$ are isospectral. If n, $n\geq 3$, i and ϕ_i are fixed then graphs $\phi_i(B_n,q)$ and $\phi_i(C_n,q)$, $q=3,5,7,9,\ldots$ form two families of isospectral regular small world graphs. Their orders grow but diameter is bounded by constant $c(n,j,\phi)$.*

Remark 1. Described above pairs of families of small world graphs are geometrical expanders (see [6]), i. e. families of regular graphs of increasing degree such that the ratio of second largest eigenvalue and degree tends to ∞ when parameter q grows.

Remark 2. We can substitute geometries $B_n(q)$ and $C_n(q)$ for totalities of their flags (cosets by parabolic subgroups).

Theorem 1 demonstrates that we not only unable to hear the shape of graph but unable to distinguish symphonies performed by two orchestras of small world graphs. We unable to distinguish by sound two different families of simple groups of Lie type acting on their geometries.