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Finite groups with given properties of normalizers of Sylow subgroups

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We consider only finite groups. We use notations and definitions from [**1**].

Let $\mathfrak{F}$ be a non-empty formation. A subgroup H is called $\mathfrak{F}$-subnormal in G, if either $H = G$, or there exists a maximal chain of subgroups $H = H_0 \leq H_1 \leq \cdots \leq H_{n-1} \leq H_n = G$ such that $H_i^{\mathfrak{F}} \leq H_{i-1}$ for $i = 1, \ldots, n$.

Recall that the class of groups $\mathrm{w}^*\mathfrak{F}$ is defined as follows:

$\mathrm{w}^*\mathfrak{F} = (G \mid \pi(G) \subseteq \pi(\mathfrak{F})$ and every normalizer of Sylow subgroup of G is $\mathfrak{F}$-subnormal in $G)$.

Theorem 1. *Let $\mathfrak{F}$ be a non-empty hereditary formation. Then the following statements are true.*

(1) $\mathfrak{F} \subseteq \mathrm{w}^*\mathfrak{F}$.

(2) $\mathrm{w}^*\mathfrak{F} = \mathrm{w}^*(\mathrm{w}^*\mathfrak{F})$.

(3) *If a formation* $\mathfrak{F}_1 \subseteq \mathfrak{F}$ *then* $\mathrm{w}^*\mathfrak{F}_1 \subseteq \mathrm{w}^*\mathfrak{F}$.

(4) $\mathrm{w}^*\mathfrak{F}$ *is a formation and from* $G \in \mathfrak{F}$ *it follows that every Hall subgroup of* G *belongs to* $\mathfrak{F}$.

According to [**2**], the arithmetic length of a soluble group G is defined as $\max\{l_p(G)\}$, where $l_p(G)$ is p-length of the group G for all $p \in \pi(G)$. Note that the class $\mathfrak{L}_a(1)$ of all soluble groups whose arithmetic length ≤ 1 is a hereditary saturated Fitting formation.

Theorem 2. *Let* $\mathfrak{F}$ *be a hereditary saturated formation and* $\mathfrak{F} \subseteq \mathfrak{L}_a(1)$. *Then* $\mathrm{w}^*\mathfrak{F} = \mathfrak{F}$.

Corollary 1. (1) [**3**] *If* $\mathfrak{N}^2$ *is the class of all metanilpotent groups, then* $\mathrm{w}^*\mathfrak{N}^2 = \mathfrak{N}^2$.

(2) [**3**] *If* $\mathfrak{NA}$ *is the class of all groups* G *with the nilpotent commutator subgroup* G', *then* $\mathrm{w}^*\mathfrak{NA} = \mathfrak{NA}$.

(3) $\mathrm{w}^*\mathfrak{L}_a(1) = \mathfrak{L}_a(1)$.

We note that $\mathrm{w}^*\mathfrak{N}^3 \neq \mathfrak{N}^3$.

References

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Cotransitive subsemigroups of the full transformation semigroup T_n

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The concept of a cotransitive subsemigroup for transformations semigroups was introduced by R.P. Sullivan in the work [**1**]. It is used to describe the ideals. We restrict ourselves to the consideration of a full transformation semigroup T_n of finite set X. For $\alpha \in T_n$ by $\pi_\alpha = \alpha \circ \alpha^{-1}$ we denote the partition of the set X into equivalence classes. Let $ran\,\alpha = \{x_1, x_2, \ldots, x_k\} \subseteq$ $\subseteq X$, $A_i = \alpha^{-1}(x_i)$. Subsemigroup $S \subseteq T_n$ is called *cotransitive*, if for every $\alpha = \begin{pmatrix} A_i \\ x_i \end{pmatrix} \in S$ with rank k we have:

(1) for every $\{b_1, b_2, \ldots, b_k\} \subseteq X$ $\mu = \begin{pmatrix} A_i \\ b_i \end{pmatrix} \in S$;

(2) for every $\{y_1, y_2, \ldots, y_k\} \subseteq X$ there exists $\lambda \in S$ such that $y_i \in \lambda^{-1}(x_i)$, $i = \overline{1,k}$.

If a cotransitive subsemigroup $S \subseteq T_n$ contains element of rank $k > 1$, then there exists such family of partitions $\{\pi_\alpha | \alpha \in S'\}$, $S' \subseteq S$ of a set X, that separates any its k elements. For $k = 1$ there is the trivial partition $\rho(1)$ with one block.

Partitions $X = \bigcup\limits_{i=1}^{k} A_i = \bigcup\limits_{i=1}^{k} B_i$ are of the *same type* if sets $(|A_1|, |A_2|, \ldots, |A_k|)$ and $(|B_1|, |B_2|, \ldots, |B_k|)$ differ only in ordering. The partition $X = \bigcup\limits_{i=1}^{k} A_i$ is called *less* than $X = \bigcup\limits_{i=1}^{r} B_i$ if every block B_i of the second partition is a union of several blocks of the first partition. We denote the lattice of all partitions of a set X by $Part\,X$.

Lemma 1. *Let $\{\rho_j(k)\}_{j\in J}$ is such family of partitions of a set X into $k > 1$ blocks, that separates any its k elements, $Q_k = \{\rho \in Part\,X \mid \rho_j(k) \le \rho$ for some $j \in J\}$. Then for $k < n$ $S = \{\alpha \in T_n \mid \pi_\alpha \in \bigcup\limits_{i=1}^{k} Q_i\}$ is cotransitive subsemigroup of semigroup T_n.*

Lemma 2. *Let $\mu_1, \mu_2, \ldots, \mu_m$ is a family of partitions of a set X into k blocks ($1 < k < n$), $\{\rho_j\}_{j\in J}$ is a family of all partitions of a set X, such that are of the same type with one of μ_i, and*