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Cotransitive subsemigroups of the full transformation semigroup T_n

Tetiana Voloshyna

The concept of a cotransitive subsemigroup for transformations semigroups was introduced by R.P. Sullivan in the work [**1**]. It is used to describe the ideals. We restrict ourselves to the consideration of a full transformation semigroup T_n of finite set X. For $\alpha \in T_n$ by $\pi_\alpha = \alpha \circ \alpha^{-1}$ we denote the partition of the set X into equivalence classes. Let $ran\, \alpha = \{x_1, x_2, \dots, x_k\} \subseteq \subseteq X$, $A_i = \alpha^{-1}(x_i)$. Subsemigroup $S \subseteq T_n$ is called *cotransitive*, if for every $\alpha = \begin{pmatrix} A_i \\ x_i \end{pmatrix} \in S$ with rank k we have:

(1) for every $\{b_1, b_2, \dots, b_k\} \subseteq X$ $\mu = \begin{pmatrix} A_i \\ b_i \end{pmatrix} \in S$;

(2) for every $\{y_1, y_2, \dots, y_k\} \subseteq X$ there exists $\lambda \in S$ such that $y_i \in \lambda^{-1}(x_i)$, $i = \overline{1,k}$.

If a cotransitive subsemigroup $S \subseteq T_n$ contains element of rank $k > 1$, then there exists such family of partitions $\{\pi_\alpha | \alpha \in S'\}$, $S' \subseteq S$ of a set X, that separates any its k elements. For $k = 1$ there is the trivial partition $\rho(1)$ with one block.

Partitions $X = \bigcup\limits_{i=1}^{k} A_i = \bigcup\limits_{i=1}^{k} B_i$ are of the *same type* if sets $(|A_1|, |A_2|, \dots, |A_k|)$ and $(|B_1|, |B_2|, \dots, |B_k|)$ differ only in ordering. The partition $X = \bigcup\limits_{i=1}^{k} A_i$ is called *less* than $X = \bigcup\limits_{i=1}^{r} B_i$ if every block B_i of the second partition is a union of several blocks of the first partition. We denote the lattice of all partitions of a set X by $Part\, X$.

Lemma 1. *Let $\{\rho_j(k)\}_{j\in J}$ is such family of partitions of a set X into $k > 1$ blocks, that separates any its k elements, $Q_k = \{\rho \in Part\, X \mid \rho_j(k) \leq \rho \text{ for some } j \in J\}$. Then for $k < n$ $S = \{\alpha \in T_n \mid \pi_\alpha \in \bigcup\limits_{i=1}^{k} Q_i\}$ is cotransitive subsemigroup of semigroup T_n.*

Lemma 2. *Let $\mu_1, \mu_2, \dots, \mu_m$ is a family of partitions of a set X into k blocks $(1 < k < n)$, $\{\rho_j\}_{j\in J}$ is a family of all partitions of a set X, such that are of the same type with one of μ_i, and*

$Q = \{\rho \in Part\, X \,|\, \rho_j \leq \rho \text{ for some } j \in J\}$. *Then* $S = S_n \bigcup \{\alpha \in T_n \,|\, \pi_\alpha \in Q\}$ *is cotransitive subsemigroup of semigroup* T_n.

Obviously, subgroup S_n is also a cotransitive subsemigroup of semigroup T_n.

Listed subsemigroups exhaust all cotransitive subsemigroups of semigroup T_n.

The following concept was introduced by I. Levi in the paper [**2**]. Subsemigroup $S \subseteq T_n$ is called S_n*–normal* if for any $g \in S_n$ $g^{-1}Sg = S$.

Theorem 1. *Cotransitive subsemigroup of semigroup* T_n *is* S_n*-normal if and only if it is a union of equivalence classes, corresponding to the same type of partition* X.

References

1. R. P. Sullivan, *Ideals in transformation semigroups*, Comment. Math. Univ. Carolin. **19** (1978), 431–446.
2. I. Levi, *Congruences on normal transformation semigroups* Math. Jap. **52** (1980), no. 2, 247–261.

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On σ-local Fitting Classes

Nikolay T. Vorob'ev

Throughout this paper all groups are finite. The notations and terminologies are standard as in [**1**].

Let σ is some partition of the set of all primes $\mathbb{P}$. If G is a finite group and $\mathfrak{F}$ is a Fitting class of finite groups, then the symbol $\sigma(G)$ denotes the set $\{\sigma_i : \sigma_i \cap \pi(|G|) \neq \emptyset\}$ and $\sigma(\mathfrak{F}) = \bigcup_{G \in \mathfrak{F}} \sigma(G)$. Following [**3**], we call any function f of the form $f : \sigma \to \{\text{Fitting class}\}$ a *Hartley* σ*-function* (or simply H_σ*-function*), and we put $LR_\sigma(f) = (G : G = 1 \text{ or } G \neq 1 \text{ and } G^{\mathfrak{E}_{\sigma_i}\mathfrak{E}_{\sigma_i'}} \in f(\sigma_i) \text{ for all } \sigma_i \in \sigma(G))$. If there is a H_σ-function f such that $\mathfrak{F} = LR_\sigma(f)$, then we say that $\mathfrak{F}$ is σ*-local* and f is a σ-local definition of $\mathfrak{F}$.

Note that in the case when $\sigma = \sigma^1 = \{\{2\}, \{3\}, \ldots\}$, σ-local Fitting class $\mathfrak{F}$ is local [**2**] and we use symbol $LR(f)$ instead $LR_\sigma(f)$. Let $\mathfrak{F} = LR_\sigma(f)$ for some H_σ-function f. Then we say that: (a) f is *integrated* if $f(\sigma_i) \subseteq \mathfrak{F}$ for all i; (b) f is *full* if $f(\sigma_i)\mathfrak{E}_{\sigma_i} = f(\sigma_i)$ for all i; (c) *full integrated* if f is full and integrated.

Recall that a Fitting class is a Lockett class, if the $\mathfrak{F}$-radical of the direct product of groups G and H is the direct product of the $\mathfrak{F}$-radical of G and $\mathfrak{F}$-radical of H for all groups G and H.

Theorem 1. *Every* σ*-local Fitting class can be defined by a unique full integrated* H_σ*-function* F *such that* $F(\sigma_i) = F(\sigma_i)\mathfrak{E}_{\sigma_i} \subseteq \mathfrak{F}$ *for all* $\sigma_i \in \sigma(\mathfrak{F})$ *and the value* $F(\sigma_i)$ *for every* $\sigma_i \in \sigma(\mathfrak{F})$ *is a Lockett class.*

Theorem 2. *Every product* $\mathfrak{F} \diamond \mathfrak{H}$ *of two* σ*-local Fitting classes* $\mathfrak{F}$ *and* $\mathfrak{H}$ *is a* σ*-local Fitting class.*

In the case when $\sigma = \sigma^1$, we get from Theorem 1 and Theorem 2 the well-known results [**2**] and [**3**] for local Fitting classes.