$Q = \{\rho \in Part\, X \mid \rho_j \le \rho \text{ for some } j \in J\}$. *Then* $S = S_n \bigcup \{\alpha \in T_n \mid \pi_\alpha \in Q\}$ *is cotransitive subsemigroup of semigroup* T_n.

Obviously, subgroup S_n is also a cotransitive subsemigroup of semigroup T_n.

Listed subsemigroups exhaust all cotransitive subsemigroups of semigroup T_n.

The following concept was introduced by I. Levi in the paper [**2**]. Subsemigroup $S \subseteq T_n$ is called S_n*–normal* if for any $g \in S_n$ $g^{-1}Sg = S$.

Theorem 1. *Cotransitive subsemigroup of semigroup* T_n *is* S_n*-normal if and only if it is a union of equivalence classes, corresponding to the same type of partition* X.

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On σ-local Fitting Classes

Nikolay T. Vorob'ev

Throughout this paper all groups are finite. The notations and terminologies are standard as in [**1**].

Let σ is some partition of the set of all primes $\mathbb{P}$. If G is a finite group and $\mathfrak{F}$ is a Fitting class of finite groups, then the symbol $\sigma(G)$ denotes the set $\{\sigma_i : \sigma_i \cap \pi(|G|) \neq \emptyset\}$ and $\sigma(\mathfrak{F}) = \bigcup_{G\in\mathfrak{F}} \sigma(G)$. Following [**3**], we call any function f of the form $f : \sigma \to \{\text{Fitting class}\}$ a *Hartley* σ*-function* (or simply H_σ*-function*), and we put $LR_\sigma(f) = (G : G = 1$ or $G \neq 1$ and $G^{\mathfrak{E}_{\sigma_i}\mathfrak{E}_{\sigma_i'}} \in f(\sigma_i)$ for all $\sigma_i \in \sigma(G))$. If there is a H_σ-function f such that $\mathfrak{F} = LR_\sigma(f)$, then we say that $\mathfrak{F}$ is σ*-local* and f is a σ-local definition of $\mathfrak{F}$.

Note that in the case when $\sigma = \sigma^1 = \{\{2\}, \{3\}, \ldots\}$, σ-local Fitting class $\mathfrak{F}$ is local [**2**] and we use symbol $LR(f)$ instead $LR_\sigma(f)$. Let $\mathfrak{F} = LR_\sigma(f)$ for some H_σ-function f. Then we say that: (a) f is *integrated* if $f(\sigma_i) \subseteq \mathfrak{F}$ for all i; (b) f is *full* if $f(\sigma_i)\mathfrak{E}_{\sigma_i} = f(\sigma_i)$ for all i; (c) *full integrated* if f is full and integrated.

Recall that a Fitting class is a Lockett class, if the $\mathfrak{F}$-radical of the direct product of groups G and H is the direct product of the $\mathfrak{F}$-radical of G and $\mathfrak{F}$-radical of H for all groups G and H.

Theorem 1. *Every* σ*-local Fitting class can be defined by a unique full integrated* H_σ*-function* F *such that* $F(\sigma_i) = F(\sigma_i)\mathfrak{E}_{\sigma_i} \subseteq \mathfrak{F}$ *for all* $\sigma_i \in \sigma(\mathfrak{F})$ *and the value* $F(\sigma_i)$ *for every* $\sigma_i \in \sigma(\mathfrak{F})$ *is a Lockett class.*

Theorem 2. *Every product* $\mathfrak{F} \diamond \mathfrak{H}$ *of two* σ*-local Fitting classes* $\mathfrak{F}$ *and* $\mathfrak{H}$ *is a* σ*-local Fitting class.*

In the case when $\sigma = \sigma^1$, we get from Theorem 1 and Theorem 2 the well-known results [**2**] and [**3**] for local Fitting classes.

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On σ-local Fitting sets

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Throughout this paper all groups are finite. The notations and terminologies are standard as in [**1**], G always denotes a group, $|G|$ is the order of G.

Let $\mathbb{P}$ be the set of all primes. If n is an integer, the symbol $\pi(n)$ denotes the set of all primes dividing n; as usual, $\pi(G) = \pi(|G|)$, the set of all primes dividing the order of G. Following [**2**], σ is a partition of $\mathbb{P}$, that is, $\sigma = \{\sigma_i : i \in I\}$, where $\mathbb{P} = \bigcup_{i\in I} \sigma_i$, $\sigma_i \bigcap \sigma_j = \emptyset$ for all $i \neq j$; $\sigma(n) = \{\sigma_i : \sigma_i \bigcap \pi(n) \neq \emptyset\}$; $\sigma(G) = \sigma(|G|)$. A set $\mathcal{F}$ of subgroups of G [**1**] is called *a Fitting set of* G if the following conditions are satisfied: i) If $T \trianglelefteq S \in \mathcal{F}$, then $T \in \mathcal{F}$; ii) If $S, T \in \mathcal{F}$ and $S, T \trianglelefteq ST$, then $ST \in \mathcal{F}$; iii) If $S \in \mathcal{F}$ and $x \in G$, then $S^x \in \mathcal{F}$. A class $\mathfrak{F}$ of groups is said *a Fitting class* [**1**] if it is closed under taking normal subgroups and products of normal $\mathfrak{F}$-subgroups. Let $\mathfrak{E}_{\sigma_i}$ be the class of all σ_i-groups and $\mathfrak{E}_{\sigma_i'}$ be the class of all σ_i'-groups.

For a Fitting set $\mathcal{F}$ of G and a Fitting class $\mathfrak{X}$ [**3**], we call the set $\{H \leq G : H/H_{\mathcal{F}} \in \mathfrak{X}\}$ of subgroups of G the product of $\mathcal{F}$ and $\mathfrak{X}$ and denote it by $\mathcal{F} \odot \mathfrak{X}$.

A function $f : \sigma \to \{\text{Fitting sets of } G\}$ *a Hartley σ-function* (or simply *H_σ-function* of G and we put

$$LFS_\sigma(f) = \{H \leq G : H = 1 \; or \; H \neq 1 \; and \; H^{\mathfrak{E}_{\sigma_i}\mathfrak{E}_{\sigma_i'}} \in f(\sigma_i) \; for \; all \; \sigma_i \in \sigma(G)\} \qquad (1)$$

Definition 1. Let $\mathcal{F}$ be a Fitting set of G. If there is an H_σ-function f such that $\mathcal{F} = LFS_\sigma(f)$, then we say that $\mathcal{F}$ is σ-local and f is a σ-local definition of $\mathcal{F}$.

If $H \leq G$, then $Fitset(H)$ will denote the intersection of all Fitting sets of G that contain H. Clearly $Fitset(H)$ is again a Fitting set of G, and so we call it the *Fitting set generated by* H. A function f of Fitting set $\mathcal{F}$ is called full, if $f(\sigma_i) = f(\sigma_i) \odot \mathfrak{E}_{\sigma_i}$ for all $\sigma_i \in \sigma(\mathcal{F})$, where $\sigma(\mathcal{F})$ is the set of all primes dividing the order of all $\mathcal{F}$-subgroups of G.

Theorem 1. *Let $\mathcal{F}$ be a σ-local Fitting set of G. Then*

(a) $\mathcal{F}$ can be defined by a unique minimal H_σ-function $\underline{f}$ such that

$$\underline{f}(\sigma_i) = Fitset(H \leq G \; : \; H = (X^{\mathfrak{E}_{\sigma_i}\mathfrak{E}_{\sigma_i'}})^x, \; X \in \mathcal{F} \; and \; x \in G) \; for \; all \; \sigma_i \in \sigma(\mathcal{F}).$$

(b) $\mathcal{F}$ can be defined by a unique full minimal H_σ-function $\underline{\underline{f}}$ such that

$$\underline{\underline{f}} = Fitset(H \leq G : H^{\mathfrak{E}_{\sigma_i}} = (X^{\mathfrak{E}_{\sigma_i}\mathfrak{E}_{\sigma_i'}})^x, X \in \mathcal{F} \; and \; x \in G) \odot \mathfrak{E}_{\sigma_i} \; for \; all \; \sigma_i \in \sigma(\mathcal{F}).$$