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On σ-local Fitting sets

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Throughout this paper all groups are finite. The notations and terminologies are standard as in [**1**], G always denotes a group, $|G|$ is the order of G.

Let $\mathbb{P}$ be the set of all primes. If n is an integer, the symbol $\pi(n)$ denotes the set of all primes dividing n; as usual, $\pi(G) = \pi(|G|)$, the set of all primes dividing the order of G. Following [**2**], σ is a partition of $\mathbb{P}$, that is, $\sigma = \{\sigma_i : i \in I\}$, where $\mathbb{P} = \bigcup_{i\in I} \sigma_i$, $\sigma_i \bigcap \sigma_j = \emptyset$ for all $i \neq j$; $\sigma(n) = \{\sigma_i : \sigma_i \bigcap \pi(n) \neq \emptyset\}$; $\sigma(G) = \sigma(|G|)$. A set $\mathcal{F}$ of subgroups of G [**1**] is called *a Fitting set of* G if the following conditions are satisfied: i) If $T \trianglelefteq S \in \mathcal{F}$, then $T \in \mathcal{F}$; ii) If $S, T \in \mathcal{F}$ and $S, T \trianglelefteq ST$, then $ST \in \mathcal{F}$; iii) If $S \in \mathcal{F}$ and $x \in G$, then $S^x \in \mathcal{F}$. A class $\mathfrak{F}$ of groups is said *a Fitting class* [**1**] if it is closed under taking normal subgroups and products of normal $\mathfrak{F}$-subgroups. Let $\mathfrak{E}_{\sigma_i}$ be the class of all σ_i-groups and $\mathfrak{E}_{\sigma_i'}$ be the class of all σ_i'-groups.

For a Fitting set $\mathcal{F}$ of G and a Fitting class $\mathfrak{X}$ [**3**], we call the set $\{H \leq G : H/H_{\mathcal{F}} \in \mathfrak{X}\}$ of subgroups of G the product of $\mathcal{F}$ and $\mathfrak{X}$ and denote it by $\mathcal{F} \odot \mathfrak{X}$.

A function $f : \sigma \to \{$Fitting sets of $G\}$ *a Hartley σ-function* (or simply *H_σ-function* of G and we put

$$LFS_\sigma(f) = \{H \leq G : H = 1 \; or \; H \neq 1 \; and \; H^{\mathfrak{E}_{\sigma_i}\mathfrak{E}_{\sigma_i'}} \in f(\sigma_i) \; for \; all \; \sigma_i \in \sigma(G)\} \qquad (1)$$

Definition 1. Let $\mathcal{F}$ be a Fitting set of G. If there is an H_σ-function f such that $\mathcal{F} = LFS_\sigma(f)$, then we say that $\mathcal{F}$ is σ-local and f is a σ-local definition of $\mathcal{F}$.

If $H \leq G$, then $Fitset(H)$ will denote the intersection of all Fitting sets of G that contain H. Clearly $Fitset(H)$ is again a Fitting set of G, and so we call it the *Fitting set generated by* H. A function f of Fitting set $\mathcal{F}$ is called full, if $f(\sigma_i) = f(\sigma_i) \odot \mathfrak{E}_{\sigma_i}$ for all $\sigma_i \in \sigma(\mathcal{F})$, where $\sigma(\mathcal{F})$ is the set of all primes dividing the order of all $\mathcal{F}$-subgroups of G.

Theorem 1. *Let $\mathcal{F}$ be a σ-local Fitting set of G. Then*

(a) $\mathcal{F}$ can be defined by a unique minimal H_σ-function $\underline{f}$ such that
$$\underline{f}(\sigma_i) = Fitset(H \leq G \,:\, H = (X^{\mathfrak{E}_{\sigma_i}\mathfrak{E}_{\sigma_i'}})^x,\; X \in \mathcal{F} \; and \; x \in G) \; for \; all \; \sigma_i \in \sigma(\mathcal{F}).$$

(b) $\mathcal{F}$ can be defined by a unique full minimal H_σ-function $\underline{\underline{f}}$ such that
$$\underline{\underline{f}} = Fitset(H \leq G : H^{\mathfrak{E}_{\sigma_i}} = (X^{\mathfrak{E}_{\sigma_i}\mathfrak{E}_{\sigma_i'}})^x,\; X \in \mathcal{F} \; and \; x \in G) \odot \mathfrak{E}_{\sigma_i} \; for \; all \; \sigma_i \in \sigma(\mathcal{F}).$$

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On the characteristic of Fischer class

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Throughout this paper, all groups are finite. In the definitions and notation, we follow [**1**]. Remind that class $\mathfrak{F}$ is a *Fitting class* if and only if the following two conditions are satisfied:
(i) If $G \in \mathfrak{F}$ and $N \trianglelefteq G$, then $N \in \mathfrak{F}$;
(ii) If $M, N \trianglelefteq G = MN$ with M and N in $\mathfrak{F}$, then $G \in \mathfrak{F}$.
For a class $\mathfrak{F}$ of groups we define:

$$\mathsf{N}_0\mathfrak{F} = (G : \exists K_i \trianglelefteq\trianglelefteq G \ (i{=}1,\ldots,r) \textit{ with } K_i \in \mathfrak{F} \textit{ and } G = < K_1,\ldots,K_r >). \quad (1)$$

A class $\mathfrak{F}$ of arbitrary groups is called a *Fischer class* if
(i) $\mathfrak{F} = \mathsf{N}_0 \quad \mathfrak{F} \neq \emptyset$, and
(ii) If $K \trianglelefteq G \in \mathfrak{F}$ and H/K is a nilpotent subgroup of G/K, then $H \in \mathfrak{F}$.

Definition 1. Let G be a group and $\mathfrak{F}$ a class of groups.
(a) We define

$$\sigma(G) = \{p\colon p \in \mathbb{P} \textit{ and } p||G|\} \quad \textit{and}$$
$$\sigma(\mathfrak{F}) = \bigcup \{\sigma(F),\ F \in \mathfrak{F}\}.$$

(b) We also define

$$Char(\mathfrak{F}) = \{p\colon p \in \mathbb{P} \textit{ and } Z_p \in \mathfrak{F}\},$$

and call Char($\mathfrak{F}$) the *characteristic* of $\mathfrak{F}$.

Let $\mathbb{P}$ be a the set of all primes, $\pi\subseteq\mathbb{P}$, $\mathfrak{N}_\pi$ and $\mathfrak{E}_\pi$ the class of all nilpotent π-groups and the class of all π-groups respectively.

Theorem 1. *Let $\mathfrak{F}$ be a Fischer class and $\pi{=}\sigma(\mathfrak{F})$. Then:*

$$1)Char(\mathfrak{F}) = \pi; \quad (2)$$

$$2)\mathfrak{N}_\pi \subseteq \mathfrak{F} \subseteq \mathfrak{E}_\pi \quad (3)$$