References

1. K. Doerk and T. Hawkes, *Finite soluble groups*, Berlin – New York : Walter de Gruyter, 1992.
2. C. Zhang and A. N. Skiba, *On Σ_t^σ-closed classes of finite groups*, Ukrainian Math. J., 70:12, (2018), 1707-1716.
3. N. Yang, W. Guo, N. T. Vorob'ev, *On $\mathfrak{F}$-injectors of Fitting set of a finite group*, Comm. Algebra, 46:1, (2018), 217-229.

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On the characteristic of Fischer class

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Throughout this paper, all groups are finite. In the definitions and notation, we follow [**1**]. Remind that class $\mathfrak{F}$ is a *Fitting class* if and only if the following two conditions are satisfied:
(i) If $G \in \mathfrak{F}$ and $N \trianglelefteq G$, then $N \in \mathfrak{F}$;
(ii) If $M, N \trianglelefteq G = MN$ with M and N in $\mathfrak{F}$, then $G \in \mathfrak{F}$.
For a class $\mathfrak{F}$ of groups we define:

$$\mathsf{N}_0\mathfrak{F} = (G : \exists K_i \trianglelefteq\trianglelefteq G\ (i{=}1, \ldots, r)\ with\ K_i \in \mathfrak{F}\ and\ G = < K_1, \ldots, K_r >). \quad (1)$$

A class $\mathfrak{F}$ of arbitrary groups is called a *Fischer class* if
(i) $\mathfrak{F} = \mathsf{N}_0 \quad \mathfrak{F} \neq \emptyset$, and
(ii) If $K \trianglelefteq G \in \mathfrak{F}$ and H/K is a nilpotent subgroup of G/K, then $H \in \mathfrak{F}$.

Definition 1. Let G be a group and $\mathfrak{F}$ a class of groups.
(a) We define

$$\sigma(G) = \{p\colon\ p \in \mathbb{P}\ and\ p||G|\}\quad and$$
$$\sigma(\mathfrak{F}) = \bigcup \{\sigma(F),\ F \in \mathfrak{F}\}.$$

(b) We also define

$$Char(\mathfrak{F}) = \{p\colon\ p \in \mathbb{P}\ and\ Z_p \in \mathfrak{F}\},$$

and call Char($\mathfrak{F}$) the *characteristic* of $\mathfrak{F}$.

Let $\mathbb{P}$ be a the set of all primes, $\pi{\subseteq}\mathbb{P}$, $\mathfrak{N}_\pi$ and $\mathfrak{E}_\pi$ the class of all nilpotent π-groups and the class of all π-groups respectively.

Theorem 1. *Let $\mathfrak{F}$ be a Fischer class and $\pi{=}\sigma(\mathfrak{F})$. Then:*

$$1) Char(\mathfrak{F}) = \pi; \quad (2)$$

$$2) \mathfrak{N}_\pi \subseteq \mathfrak{F} \subseteq \mathfrak{E}_\pi \quad (3)$$

We shown that if $\mathfrak{F}$ is not a Fischer class, then the conditions (2) and (3) theorem 1 are not true.

References

1. K. Doerk and T. Hawkes, *Finite soluble groups*, Berlin - New York : Walter de Gruyter, 1992.

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On adjoint groups of radical rings

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An associative algebra R without identity is called radical if the set of its elements forms a group with respect to the operation $a \circ b = a + b + ab$ and R is nilpotent if $R^n = 0$ for some positive integer n. It is well-known that every nilpotent algebra is radical and the set of elements of R forms a group with respect to the operation $a \circ b = a + b + ab$ with $a,\ b \in R$. This group is called the adjoint group of R and is denoted by R°. Obviously any subalgebra of R is a subgroup of R°, but the converse is not true.

Radical algebras whose all subgroups of their adjoint groups are subalgebras were described in [**1**]. Recall also that a finite group G is said to be a Miller–Moreno group if G is non-abelian and all proper subgroups of G are abelian. The following assertion is proved in [**2**], Lemma 3.3.

Lemma 1. *Let a Miller–Moreno p-group G be the adjoint group of a nilpotent p-algebra. Then one of the following statements holds:*

1) *G is a metacyclic 2-group of order at most 16;*
2) *G is a non-metacyclic 2-group of exponent 4 and of order at most 32;*
3) *G is a non-abelian p-group of order p^3 and exponent p for odd p.*

Using this lemma and the description of radical algebras given in [**1**], the following statement can be verified.

Proposition 1. *If a Miller–Moreno p-group G is the adjoint group of a nilpotent algebra R, then every subgroup of G is a subalgebra in R.*

It was proved in [**3**], Theorem 4.3, that every radical ring and in particular algebra whose adjoint group is generated by two elements is nilpotent. From this and Proposition 1 the following result is derived.

Theorem 1. *Let R be a radical algebra over a field of prime characteristic p. Then the following statements are equivalent:*

1) *every subgroup of the adjoint group R° is a subalgebra in R;*
2) *every abelian subgroup of the adjoint group R° is a subalgebra in R;*