We shown that if $\mathfrak{F}$ is not a Fischer class, then the conditions (2) and (3) theorem 1 are not true.

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On adjoint groups of radical rings

Vladislav G. Yurashev

An associative algebra R without identity is called radical if the set of its elements forms a group with respect to the operation $a \circ b = a + b + ab$ and R is nilpotent if $R^n = 0$ for some positive integer n. It is well-known that every nilpotent algebra is radical and the set of elements of R forms a group with respect to the operation $a \circ b = a + b + ab$ with $a,\ b \in R$. This group is called the adjoint group of R and is denoted by R°. Obviously any subalgebra of R is a subgroup of R°, but the converse is not true.

Radical algebras whose all subgroups of their adjoint groups are subalgebras were described in [**1**]. Recall also that a finite group G is said to be a Miller–Moreno group if G is non-abelian and all proper subgroups of G are abelian. The following assertion is proved in [**2**], Lemma 3.3.

Lemma 1. *Let a Miller–Moreno p-group G be the adjoint group of a nilpotent p-algebra. Then one of the following statements holds:*

1) *G is a metacyclic 2-group of order at most 16;*
2) *G is a non-metacyclic 2-group of exponent 4 and of order at most 32;*
3) *G is a non-abelian p-group of order p^3 and exponent p for odd p.*

Using this lemma and the description of radical algebras given in [**1**], the following statement can be verified.

Proposition 1. *If a Miller–Moreno p-group G is the adjoint group of a nilpotent algebra R, then every subgroup of G is a subalgebra in R.*

It was proved in [**3**], Theorem 4.3, that every radical ring and in particular algebra whose adjoint group is generated by two elements is nilpotent. From this and Proposition 1 the following result is derived.

Theorem 1. *Let R be a radical algebra over a field of prime characteristic p. Then the following statements are equivalent:*

1) *every subgroup of the adjoint group R° is a subalgebra in R;*
2) *every abelian subgroup of the adjoint group R° is a subalgebra in R;*

3) *every non-abelian subgroup of the adjoint group R° is a subalgebra in R.*

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Commutative Bezout ring, which is a ring of neat range 1

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All rings considered will be commutative with nonzero unit.

Recall that ring is Bezout ring if it finitely generated ideals is principal. Ring R is said to have a stable range 2 if for every elements $a, b, c \in R$ such that $aR + bR + cR = R$ we have $(a + cx)R + (b + cy)R = R$ for some elements $x, y \in R$. Ring R is called an elementary divisor ring if for any matrix A of order $n \times m$ over R there exist invertible matrices $P \in GL_n(R)$ and $Q \in GL_m(R)$ such that $PAQ = D$ is a diagonal matrix, $D = (d_{ii})$ and $d_{i+1,i+1}R \subset d_{ii}R$. A ring R is called a clean ring if for any $a \in R$ there exist invertible element $u \in R$ and idempotent $e \in R$ such that $a = e + u$. Element $a \in R$ is called a neat element if factor-ring R/aR is a clean ring. Ring R is called a ring of neat range 1 if from condition $aR + bR = R$ implies that $a + bt$ is a neat element for some $t \in R$.

Proposition 1. *Let R be a commutative Bezout ring of neat range 1. Then for any ideal I of R factor-ring R/I is a ring of neat range 1.*

Proposition 2. *A commutative Bezout ring is a ring of neat range 1 if and only if factor-ring $R/J(R)$ is a ring of neat range 1 (where $J(R)$ – is Jacobson radical).*

Theorem 1. *Commutative Bezout ring in which all zero divisors are in Jacobson radical is an elementary divisor ring if and only if it is a ring of neat range 1.*

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