3) *every non-abelian subgroup of the adjoint group R° is a subalgebra in R.*

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Key words and phrases. Nilpotent algebra, radical algebra, Miller–Moreno group, adjoint group

Commutative Bezout ring, which is a ring of neat range 1

Bohdan Zabavskyi, Olha Domsha

All rings considered will be commutative with nonzero unit.

Recall that ring is Bezout ring if it finitely generated ideals is principal. Ring R is said to have a stable range 2 if for every elements $a, b, c \in R$ such that $aR + bR + cR = R$ we have $(a + cx)R + (b + cy)R = R$ for some elements $x, y \in R$. Ring R is called an elementary divisor ring if for any matrix A of order $n \times m$ over R there exist invertible matrices $P \in GL_n(R)$ and $Q \in GL_m(R)$ such that $PAQ = D$ is a diagonal matrix, $D = (d_{ii})$ and $d_{i+1,i+1}R \subset d_{ii}R$. A ring R is called a clean ring if for any $a \in R$ there exist invertible element $u \in R$ and idempotent $e \in R$ such that $a = e + u$. Element $a \in R$ is called a neat element if factor-ring R/aR is a clean ring. Ring R is called a ring of neat range 1 if from condition $aR + bR = R$ implies that $a + bt$ is a neat element for some $t \in R$.

Proposition 1. *Let R be a commutative Bezout ring of neat range 1. Then for any ideal I of R factor-ring R/I is a ring of neat range 1.*

Proposition 2. *A commutative Bezout ring is a ring of neat range 1 if and only if factor-ring $R/J(R)$ is a ring of neat range 1 (where $J(R)$ – is Jacobson radical).*

Theorem 1. *Commutative Bezout ring in which all zero divisors are in Jacobson radical is an elementary divisor ring if and only if it is a ring of neat range 1.*

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Key words and phrases. An elementary divisor ring, a ring of neat range 1

J-Noetherian Bezout domain which are not of stable range 1

Bohdan Zabavsky, Oleh Romaniv

All rings considered will be commutative and have identity.

A ring R is a ring of stable range 1 if for any $a, b \in R$ such that $aR + bR = R$ we have $(a + bt)R = R$ for some $t \in R$.

An element a is an element of stable range 1 if for any $b \in R$ such that $aR + bR = R$ we have $a + bt$ is an invertible element for some $t \in R$.

An element $a \in R$ is an element of almost stable range 1 if R/aR is a ring of stable range 1.

By a Bezout ring we mean a ring in which all finitely generated ideals are principal.

By a J-ideal of R we mean an intersection of maximal ideals of R.

A ring R is J-Noetherian provided R has maximum condition of J-ideals.

A commutative ring R is called an elementary divisor ring [**3**] if for an arbitrary matrix A of order $n \times m$ over R there exist invertible matrices $P \in GL_n(R)$ and $Q \in GL_m(R)$ such that

$PAQ = D$ is diagonal matrix, $D = (d_{ii})$,

$d_{i+1,i+1}R \subset d_{ii}R$.

Let R be a Bezout domain. An element $a \in R$ is called a neat element if for every elements $b, c \in R$ such that $bR + cR = R$ there exist $r, s \in R$ such that $a = rs$ where $rR + bR = R$, $sR + cR = R$ and $rR + sR = R$. A Bezout domain is said to be of neat range 1 if for any $c, b \in R$ such that $cR + bR = R$ there exists $t \in R$ such that $a + bt$ is a neat element.

Theorem 1. *A commutative Bezout domain R is an elementary divisor domain if and only if R is a ring of neat range 1.*

Theorem 2. *A nonunit divisor of a neat element of a commutative Bezout domain is a neat element.*

Theorem 3. *Let R be a J-Noetherian Bezout domain which is not a ring of stable range 1. Then in R there exists an element $a \in R$ such that R/aR is a local ring.*

By [**8**], any adequate element of a commutative Bezout ring is a neat element. An element a of a domain R is said to be adequate, if for every element $b \in R$ there exist elements $r, s \in R$ such that (1) $a = rs$; (2) $rR + bR = R$ (3) $\hat{s}R + bR \neq R$ for any $\hat{s} \in R$ such that $sR \subset \hat{s}R \neq R$. A domain R is called adequate if every nonzero element of R is adequate [**4**].

Theorem 4. *Let R be a commutative Bezout element and a is non-zero nonunit element of R. If R/aR is local ring, then a is an adequate element.*

Theorem 5. *Let R be a J-Noetherian Bezout domain which is not a ring of stable range 1. Then in R there exists a nonunit adequate element.*

Theorem 6. *Let R be a Bezout domain in which every nonzero nonunit element has only finitely many prime ideals minimal over it. Then the factor ring R/aR is the finite direct sum of valuation rings.*