Permutation Kirichenko's Latins square

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One of the most important classes, which appear in various questions of the ring theory and the image theory, is the class of the tiled orders. Each tiled order is completely determined by its exponent matrix and discrete valuation ring. Many of the properties of these rings are completely determined by their exponent matrix, such as quivers of rings. We continues the study of exponent matrices. It is devoted to research of exponent matrices that are Latin squares and their quivers. We found all possible Kirichenko's permutation for Gorenstein matrices which are Latin squares.

Theorem 1. *Gorenstein matrix can not be a Latin square in two cases*

1) *Decomposition permutation Kirichenko of Gorenstein matrix on independent cycles contains cycles of different lengths.*
2) *Decomposition permutation Kirichenko of Gorenstein matrix on independent cycles contain an even number of cycles of odd length.*

For other permutation Kirichenko exist Gorenstein Latin squares.

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On equivalence and factorization of the Kronecker product of matrices

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Let R be a principal ideal ring, $A \in M_m(R)$, $B \in M_n(R)$. The Kronecker product of matrices A and B will be denoted by $A \otimes B = (a_{ij}B)$ (see [3]).

Theorem 1. *If the matrix A is equivalent to the matrix A_1 and $A \otimes B$ is equivalent to $A_1 \otimes B_1$ then the matrix B is equivalent to the matrix B_1.*

Let $A(x) \in M_m(F[x])$ and $B(x) \in M_n(F[x])$ be the polynomial matrices and F be an algebraically closed field of characteristic zero, that is

$$A(x) = \sum_{i=0}^{s_1} A_i x^{s_1-i}, \quad B(x) = \sum_{i=0}^{s_2} B_i x^{s_2-i}.$$

Using the books [1] and [2] we get the following results.

THEOREM 2. *If $A(x)$ and $B(x)$ are the regular polynomial matrices of a simple structure and $(det(A(x), detB(x)) = 1$, then*

$$A(x) \otimes B(x) = (A_0 \otimes B_0)(E_{mn}x - C_1)(E_{mn}x - C_2)\dots(E_{mn}x - C_{s_1+s_2}),$$

where $(E_{mn}x - C_i)$ are the matrices of a simple structure.

THEOREM 3. *Let $A(x)$ and $B(x)$ are the regular polynomial matrices and not more than one of elementary divisor of one of them is of degree two and the rest are degrees of not more than one. Then the regular polynomial matrix $A(x) \otimes B(x)$ is decomposed into a product of linear regular factors.*

To obtain such factorizations, matrices $M_{G(x)}(\varphi_k)$ are used, $G(x)$ are based on matrix $A(x) \otimes B(x)$, $\varphi_k(x)$ is divisor of degree mn of polynomial $(detA)^n(detB)^m$.

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The least dimonoid congruences on the free n-nilpotent trioid

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Motivated by problems of algebraic topology, J.-L. Loday and M.O. Ronco introduced the notion of a trioid [1]. The notion of a dimonoid was introduced in [2].

If ρ is a congruence on a trioid $(T, \dashv, \vdash, \perp)$ such that two operations of $(T, \dashv, \vdash, \perp)/\rho$ coincide and it is a dimonoid, we say that ρ is a dimonoid congruence [**3**]. A dimonoid congruence ρ on a trioid $(T, \dashv, \vdash, \perp)$ is called a $d^{\perp}_{\dashv}$-congruence (respectively, $d^{\perp}_{\vdash}$-congruence) [**3**] if operations $\dashv$ and $\perp$ (respectively, $\vdash$ and $\perp$) of $(T, \dashv, \vdash, \perp)/\rho$ coincide. If ρ is a congruence on a trioid $(T, \dashv, \vdash, \perp)$ such that all operations of $(T, \dashv, \vdash, \perp)/\rho$ coincide, we say that ρ is a semigroup congruence.