Let $A(x) \in M_m(F[x])$ and $B(x) \in M_n(F[x])$ be the polynomial matrices and F be an algebraically closed field of characteristic zero, that is

$$A(x) = \sum_{i=0}^{s_1} A_i x^{s_1-i}, \quad B(x) = \sum_{i=0}^{s_2} B_i x^{s_2-i}.$$

Using the books [1] and [2] we get the following results.

Theorem 2. *If $A(x)$ and $B(x)$ are the regular polynomial matrices of a simple structure and $(det(A(x), detB(x)) = 1$, then*

$$A(x) \otimes B(x) = (A_0 \otimes B_0)(E_{mn}x - C_1)(E_{mn}x - C_2) \dots (E_{mn}x - C_{s_1+s_2}),$$

where $(E_{mn}x - C_i)$ are the matrices of a simple structure.

Theorem 3. *Let $A(x)$ and $B(x)$ are the regular polynomial matrices and not more than one of elementary divisor of one of them is of degree two and the rest are degrees of not more than one. Then the regular polynomial matrix $A(x) \otimes B(x)$ is decomposed into a product of linear regular factors.*

To obtain such factorizations, matrices $M_{G(x)}(\varphi_k)$ are used, $G(x)$ are based on matrix $A(x) \otimes B(x)$, $\varphi_k(x)$ is divisor of degree mn of polynomial $(detA)^n(detB)^m$.

References

1. P.S. Kazimirs'kii, *Factorizations of matrix polynomials*, Pidstryhach Institute for Applied Problems of Mechanics and Mathematics NAS of Ukraine, L'viv, 2015.
2. Vasyl' Petrychkovych, *Generalized equivalence of matrices and its collections and factorization of matrices over rings*, Pidstryhach Institute for Applied Problems of Mechanics and Mathematics NAS of Ukraine, L'viv, 2015.
3. Morris Newman, *A Property of Equivalence*, J. Res. Nat. Bur. Stand. (U.S.) **78B** (Math. Sci.), (1974), no. 2, 71–72.

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The least dimonoid congruences on the free n-nilpotent trioid

Anatolii V. Zhuchok

Motivated by problems of algebraic topology, J.-L. Loday and M.O. Ronco introduced the notion of a trioid [1]. The notion of a dimonoid was introduced in [2].

If ρ is a congruence on a trioid $(T, \dashv, \vdash, \perp)$ such that two operations of $(T, \dashv, \vdash, \perp)/\rho$ coincide and it is a dimonoid, we say that ρ is a dimonoid congruence [**3**]. A dimonoid congruence ρ on a trioid $(T, \dashv, \vdash, \perp)$ is called a $d^{\perp}_{\dashv}$-congruence (respectively, $d^{\perp}_{\vdash}$-congruence) [**3**] if operations $\dashv$ and $\perp$ (respectively, $\vdash$ and $\perp$) of $(T, \dashv, \vdash, \perp)/\rho$ coincide. If ρ is a congruence on a trioid $(T, \dashv, \vdash, \perp)$ such that all operations of $(T, \dashv, \vdash, \perp)/\rho$ coincide, we say that ρ is a semigroup congruence.

As usual, $\mathbb{N}$ denotes the set of all positive integers. For any $n, k \in \mathbb{N}$ and $L \subseteq \{1, 2, \ldots, n\}$, $L \neq \emptyset$, we let $L + k = \{m + k \mid m \in L\}$.

Let X be an arbitrary nonempty set, and let w be an arbitrary word over the alphabet X. The length of w is denoted by ℓ_w. Let $F[X]$ be the free semigroup on X. Fix $n \in \mathbb{N}$. Define operations $\dashv$, $\vdash$, and $\perp$ on

$$FNT_n = \{(w, L) \mid w \in F[X], \ell_w \leq n,\ L \subseteq \{1, 2, ..., \ell_w\},\ L \neq \emptyset\} \cup \{0\}$$

by

$$(w, L) \dashv (u, R) = \begin{cases} (wu, L), & \ell_{wu} \leq n, \\ 0, & \ell_{wu} > n, \end{cases}$$

$$(w, L) \vdash (u, R) = \begin{cases} (wu, R + \ell_w), & \ell_{wu} \leq n, \\ 0, & \ell_{wu} > n, \end{cases}$$

$$(w, L) \perp (u, R) = \begin{cases} (wu, L \cup (R + \ell_w)), & \ell_{wu} \leq n, \\ 0, & \ell_{wu} > n, \end{cases}$$

$$(w, L) * 0 = 0 * (w, L) = 0 * 0 = 0$$

for all $(w, L), (u, R) \in FNT_n \backslash \{0\}$ and $* \in \{\dashv, \vdash, \perp\}$. The algebra $(FNT_n, \dashv, \vdash, \perp)$ will be denoted by $FNT_n(X)$.

Lemma 1. *$FNT_n(X)$ is a trioid.*

The free n-nilpotent trioid $P_n^0(X)$ was first constructed in [**4**].

Theorem 1. *The free n-nilpotent trioid $P_n^0(X)$ is isomorphic to the trioid $FNT_n(X)$.*

We characterize the least dimonoid congruences and the least semigroup congruence on $FNT_n(X)$ and consider separately free n-nilpotent trioids of rank 1.

References

1. J.-L. Loday, M.O. Ronco, *Trialgebras and families of polytopes.* Contemp. Math. **346** (2004), 369–398.
2. J.-L. Loday, *Dialgebras.* In: Dialgebras and related operads: Lect. Notes Math., Berlin: Springer-Verlag **1763** (2001), 7–66.
3. A.V. Zhuchok, *Free commutative trioids.* Semigroup Forum **98** (2019), no. 2, 355–368.
4. Yul.V. Zhuchok, *Free n-nilpotent trioids.* Mat. Stud. **43** (2015), no. 1, 3–11.

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On the structure of free trioids

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Trioids were introduced by J.-L. Loday and M.O. Ronco in the context of algebraic topology [1]. A trialgebra [1] is just a linear analog of a trioid. For extensive information on trioids see [2]. The construction of the free monogenic trioid was presented in [1]. In [3] decompositions of free trioids into tribands and bands of subtrioids were characterized and the least rectangular