As usual, $\mathbb{N}$ denotes the set of all positive integers. For any $n, k \in \mathbb{N}$ and $L \subseteq \{1, 2, \ldots, n\}$, $L \neq \emptyset$, we let $L + k = \{m + k \mid m \in L\}$.

Let X be an arbitrary nonempty set, and let w be an arbitrary word over the alphabet X. The length of w is denoted by ℓ_w. Let $F[X]$ be the free semigroup on X. Fix $n \in \mathbb{N}$. Define operations $\dashv$, $\vdash$, and $\perp$ on

$$FNT_n = \{(w, L) \mid w \in F[X], \ell_w \leq n,\ L \subseteq \{1, 2, ..., \ell_w\},\ L \neq \emptyset\} \cup \{0\}$$

by

$$(w, L) \dashv (u, R) = \begin{cases} (wu, L), & \ell_{wu} \leq n, \\ 0, & \ell_{wu} > n, \end{cases}$$

$$(w, L) \vdash (u, R) = \begin{cases} (wu, R + \ell_w), & \ell_{wu} \leq n, \\ 0, & \ell_{wu} > n, \end{cases}$$

$$(w, L) \perp (u, R) = \begin{cases} (wu, L \cup (R + \ell_w)), & \ell_{wu} \leq n, \\ 0, & \ell_{wu} > n, \end{cases}$$

$$(w, L) * 0 = 0 * (w, L) = 0 * 0 = 0$$

for all $(w, L), (u, R) \in FNT_n \backslash \{0\}$ and $* \in \{\dashv, \vdash, \perp\}$. The algebra $(FNT_n, \dashv, \vdash, \perp)$ will be denoted by $FNT_n(X)$.

Lemma 1. *$FNT_n(X)$ is a trioid.*

The free n-nilpotent trioid $P_n^0(X)$ was first constructed in [**4**].

Theorem 1. *The free n-nilpotent trioid $P_n^0(X)$ is isomorphic to the trioid $FNT_n(X)$.*

We characterize the least dimonoid congruences and the least semigroup congruence on $FNT_n(X)$ and consider separately free n-nilpotent trioids of rank 1.

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Contact information

Anatolii V. Zhuchok
Department of Algebra and System Analysis, Luhansk Taras Shevchenko National University, Starobilsk, Ukraine
Email address: zhuchok.av@gmail.com

Key words and phrases. Trioid, free n-nilpotent trioid, dimonoid, congruence

On the structure of free trioids

Yuliia V. Zhuchok

Trioids were introduced by J.-L. Loday and M.O. Ronco in the context of algebraic topology [1]. A trialgebra [1] is just a linear analog of a trioid. For extensive information on trioids see [2]. The construction of the free monogenic trioid was presented in [1]. In [3] decompositions of free trioids into tribands and bands of subtrioids were characterized and the least rectangular band congruence, the least left zero congruence and the least right zero congruence on the free trioid were presented. In this work we continue to study the structural properties of free trioids.

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Contact information

Yuliia V. Zhuchok
Department of Algebra and System Analysis, Luhansk Taras Shevchenko National University, Starobilsk, Ukraine
Email address: yulia.mih1984@gmail.com

Key words and phrases. Trioid, free trioid

Representations of ordered doppelsemigroups

Yurii V. Zhuchok

A *doppelsemigroup* [1] is an algebraic system consisting of a nonempty set D with two binary associative operations $\dashv$ and $\vdash$ satisfying the following identities

$$(D_1) \quad (x\dashv y)\vdash z = x\dashv(y\vdash z),$$
$$(D_2) \quad (x\vdash y)\dashv z = x\vdash(y\dashv z).$$

Let $(D, \dashv, \vdash)$ be an arbitrary doppelsemigroup and let $\leq$ be a partial order relation on D. The algebraic system $(D, \dashv, \vdash, \leq)$ is called an *ordered doppelsemigroup* [2] if the order relation $\leq$ is stable with respect to both operations $\dashv$ and $\vdash$.

In [2] it was proved that any ordered doppelsemigroup can be embedded to a suitable ordered doppelsemigroup consisiting of binary relations. Here we construct new ordered doppelsemigroups and study other representations of ordered doppelsemigroups.

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Contact information

Yurii V. Zhuchok
Department of Algebra and System Analysis, Luhansk Taras Shevchenko University, Starobilsk, Ukraine
Email address: zhuchok.yu@gmail.com

Key words and phrases. Doppelsemigroup, ordered doppelsemigroup, representation