Variants of absolute direct summand property

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A right module M over a unital ring R is said to be *absolute direct summand* (for short ADS) provided $M = S \oplus T'$ for every submodules S, T, T' such that $M = S \oplus T$ and T' is a complement of S. However the concept was introduced already by Laslo Fuchs in late 60's, only recent results show importance of the notion for research of relative injectivity in module categories and for structure of corresponding rings [**1**]. The natural generalization of the notion provided by restriction on a choice of T' in the definition to some particular classes of submodules appears to be a useful tool for further study of injectivity properties of modules and rings. In particular, M is called an essentially ADS-module if $M = S \oplus T'$ for each decomposition $M = S \oplus T$ and each complement T' of S with $T' \cap T = 0$ and $S \cap (T' \oplus T) \leq^e S$, and M is *type ADS* if $M = S \oplus T'$ for each decomposition $M = S \oplus T$ with type submodules S, T and each type complement T' of S.

We will discuss possible generalization of the following basic structural results about variants of ADS-modules:

Theorem 1. *Let M be a module*

[**1**, Proposition 3.2] *M is ADS if and only if A and B are mutually injective modules for every decomposition $M = A \oplus B$.*

[**2**, Theorem 2.5] *M is type ADS if and only if A and B are mutually injective modules for every type decomposition $M = A \oplus B$.*

[**3**, Theorem 2.10] *M is essentially ADS if and only if A and B are automorphism invariant modules and $A \cong B$ for every decomposition $M = A \oplus B$ such that $E(A) \cong E(B)$.*

The main goal of the talk is to describe non-singular rings R which are e-ADS or type ADS as right modules R_R:

Theorem 2. *Let R be a right non-singular ring and Q be its maximal right ring of quotients.*

[**2**, Theorem 3.3] *The module R_R is type ADS.*

[**3**, Theorem 4.11] *The module R_R is essentially ADS if and only if either $eQ \ncong (1-e)Q$ for any idempotent $e \in R$ or $R_R \cong M_2(S)$ for a suitable right automorphism invariant ring S.*

References

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Key words and phrases. ADS, essentially ADS, type ADS modules and rings, automorphism invariant modules